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Thin Film Flow on an Inclined Channel

Leo Hari Wiryanto

Faculty of Mathematics and Natural Sciences, Bandung Institute of Technology, Jalan Gensha 10 Bandung, Indonesia

<leo@math.itb.ac.id>

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Abstract. A single equation of thin film flow is modeled from lubrication theory. The model is then solved numerically to simulate wave propagation. The fluid viscosity and surface tension are involved in the formulation together with the angle of the bottom, so that it can be observed the deformation of the surface profile from sinusoidal to roll wave, i.e. typical wave having almost shock for the front edge. The surface tension effects to the deformation when the slope of the channel is relatively high.

1. Introduction

Fluid flow on an inclined channel can be found in daily life, such as on an open channel and also windscreen. Both have different character based on the thickness of the fluid. Water flowing in a river has relatively deep fluid, so that the diffusion terms cannot be neglected. That is the difference with the thin film occurred at the windscreen. Therefore, the Navier-Stokes equations for thin film flow can be reduced into lubrication theory.

In the previous International Conference on Technology (2017), Wiryanto [1] presented the model of thin film flow. Numerical solution of the model indicated that the surface wave propagated by decreasing the amplitude. Wiryanto and Febrianti [2] derived the model based on the continuity equation and the momentum equation from lubrication theory, without involving pressure distribution from the inside of the fluid at the surface. Those governing equations were then formulated into a single equation, that involved the boundary conditions along the bottom and the free surface. However, they assumed that the pressure was hydrostatic and chosen zero along the free surface. King, et. al. [3] modelled the similar problem for steady flow but involving upper layer of the air flow. The model was also a single equation in form of an integro-differential equation. Therefore, in this paper, we develop the model in [1] by involving the pressure distribution from the inside of the fluid, expressing to the curvature of the surface, and the surface tension plays an important role in the model.

Since the model is strongly nonlinear, numerical approach is proposed to be chosen for solving the model. A finite difference method is developed following [1], as it has been analyzed the stability by Wiryanto [4]. The method is then used to simulate the wave propagation. The sinusoidal wave deforms into shock-like wave. Fauzan and Wiryanto [5] found similar wave type but from shallow water equations. How fast this type of wave can be formed, it depends on the angle of the bottom. Other works related to shock-like wave and roll-wave can be seen in [6,7,8].

2. Formulation

The sketch of the flow and coordinates are shown in Fig. 1. The fluid flows on an inclined channel of angle θ. The fluid thickness is h measured from the bottom of the channel. The horizontal x −axis is chosen along the bottom of the channel and the vertical y −axis is perpendicular to x −axis, so that the gravity is projected to those axis effected to the movement of the fluid particle.

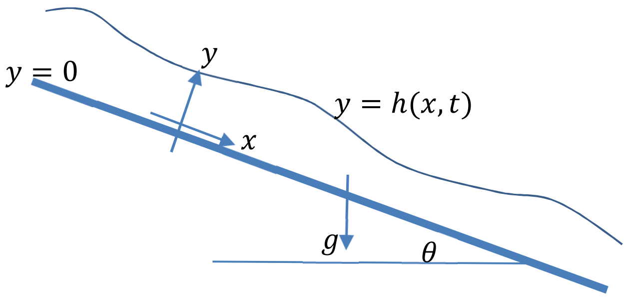

Fig. 1. Sketch of the flow and coordinates

Following Wiryanto and Febrianti [2], the governing equations can be formulated into the film thickness h satisfying

$$h_t + \frac{\rho}{3\mu}\left[h^3\left(-h_x g\cos\theta + g\sin\theta + \frac{\gamma}{\rho}h_{xxx}\right)\right]_x = 0 \qquad (1)$$

where ρ is the fluid density, μ is viscosity, γ is the surface tension and g is the acceleration of gravity. The last term represents the pressure at the surface as the effect of the surface tension, but the atmospheric pressure is chosen as the reference along the surface. Wiryanto and Febrianti [1] neglect that term.

Since the model is strongly nonlinear, a numerical approach is used. To reduce the derivative, the equation is firstly integrated with respect to x, on a small interval $[x_i, x_{i+1}]$, and then the integration of h_t is approximated by trapezoidal rule. Meanwhile, the value for h^3, h_x, h_{xxx} at (x_i, t_n) is approximated by

$$h^3 \approx \left(\frac{h_i^n + h_{i+1}^n}{2}\right)^3$$

$$h_x \approx \frac{h_{i+1}^n - h_i^n}{\triangle x}$$

$$h_{xxx} \approx \frac{h_{i+2}^n - 3h_{i+1}^n + 3h_i^n - h_{i-1}^n}{\triangle x^3}$$

so that the finite difference equation of (1) is presented in explicit form. The numerical procedure is developed from Wiryanto & Febrianti [2], by involving third derivative of h.

3. Numerical Results

In this section, the numerical solution of the model (1) is presented. Most of the calculations uses $g = 10$, $\rho = 1$, and various values μ and angle θ. As the discretization, it uses $\Delta x = 0.1$ and the step time is $\Delta t = 0.01$.

Figure 2 is typical solution of (1) presenting surface profile of thin film flow on an inclined channel, without involving surface tension. The profile deforms from sinusoidal to almost shock at the front of wave. The deformation is shown by plotting some surfaces at different time as indicated following t-axis, instead of the surface animation. The solution was calculated by using initial value $h(x,0) = 0.1 + 0.03\ \sin(0.06\ \pi\ x)$. The bottom of the channel is inclined by angle $\theta = 20$ degree. This is able to shift the crest of the wave moving faster than trough.

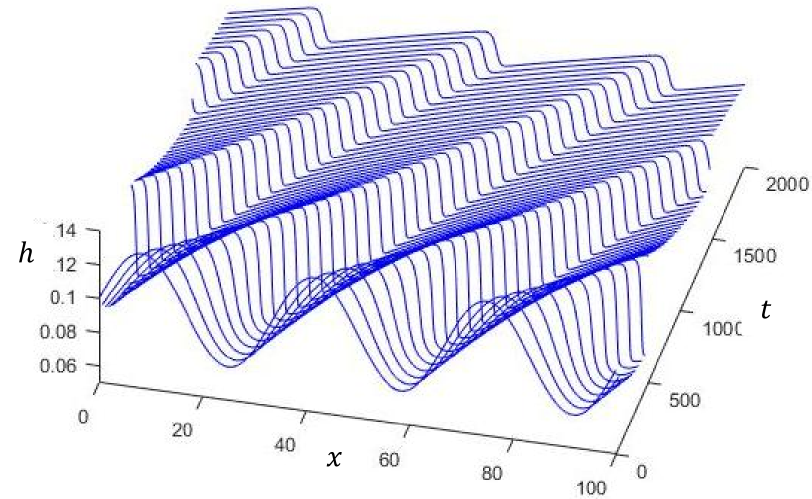

Fig. 2. Plot of the surface profile of thin film flow without involving surface tension. This plot animates the deformation from sinusoidal form to almost shock.

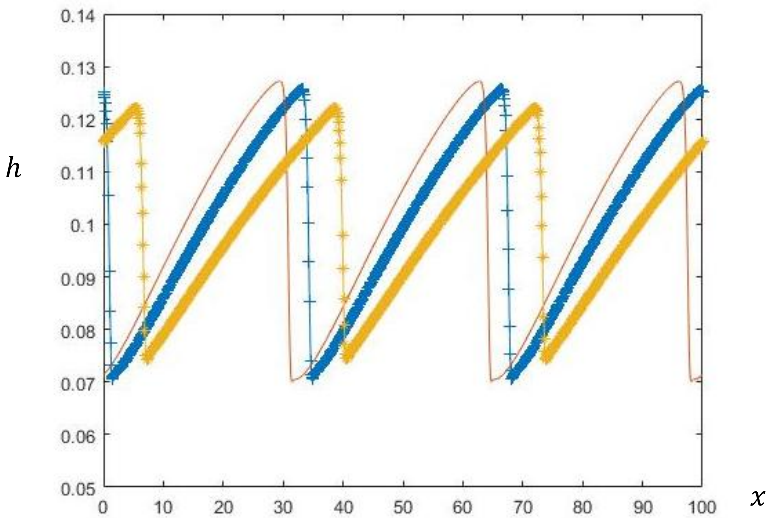

Fig. 3. Plot of the surface profile $h(x, 2000)$ without involving surface tension, for different viscosity, $\mu = 1$ given the plot in "-", $\mu = 0.8$ in "+-" and $\mu = 0.6$ in "*-"

From the numerical calculation, it can be observed that the viscosity reduces the wave speed when we increase μ. We present the surface h of the fluid in Figure 3 as the result of different viscosity, $\mu = 1, 0.8$ and 0.6 corresponding to plot in "-", "+-" and "*-" respectively. Those plots are obtained from the same initial value as in Figure 2, and calculated until $t = 2000$. It can be seen the comparison, smaller viscosity produces wave with reducing amplitude and moving faster.

In Figure 4, the effect of the surface tension is shown. The third derivation of h with respect of x is involved in the calculation by giving the parameter of the surface tension $\gamma = 0.4$. This can produce the deformation of sinusoidal wave into sharp front wave with appearing cusp-like in front and behind the front wave. This form does not appear when $\gamma = 0$, compare to the plot in Figure 2.

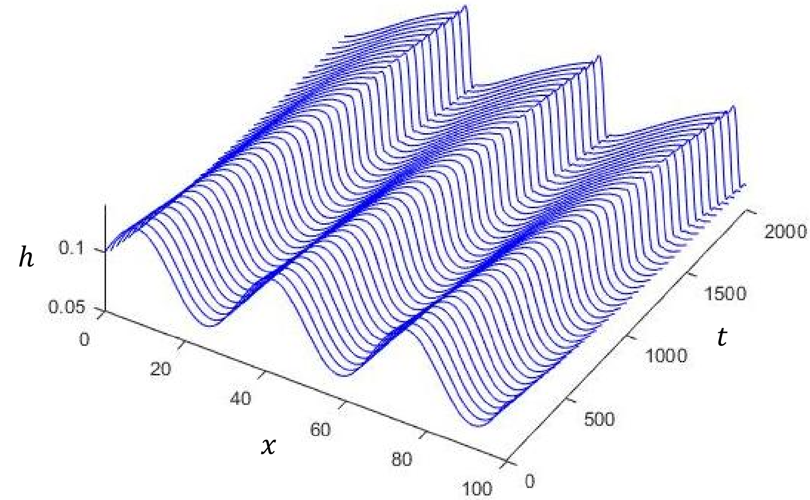

Fig. 4. Plot of some surfaces $h(x, t)$ for different time t as the solution of (1) by involving surface tension $\gamma = 0.4$

4. Conclusion

A model of thin film flow has been solved numerically by involving surface tension. The method was a finite difference, that was stable, so that it could be observed to the wave deformation. The sinusoidal wave was chosen as the initial value and it deformed to almost shock wave at the front wave. When the surface tension is involved the profile around the front wave is slightly different by appearing cusp-like at the crest and trough.

Acknowledgements

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