

## **Motor control algorithm with advantages of PID method and LMS method**

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**Abstract.** PID control method is widely used in practical control and has a quick response performance. On the other hand, the adaptive control method has the automatic modification and can adapt for the change of control environments. In this paper, a control system and an algorithm to consistent these advantages are proposed.

### **1. Introduction**

PID controller has a simple structure configured by Proportional (P) controller, Integral (I) controller and Differential (D) controller, and the role of these controllers is distinct. In addition, PID control has a quick response performance required in the practical control. From these reasons, PID control is widely used at the motor control, and many tuning methods are given [1, 2]. However, the suitable PID parameters are affected by control environment, e.g. motor load and temperature. In practice, control environments are not necessarily constant. Therefore, it is desirable that the controller parameter is modified automatically for the change of control environments.

One of the methods to realize this requirement is the adaptive filtering control using LMS (Least Mean Square) method. LMS method is the algorithm to solve gradient descent approximately, that is, to calculate the minimum of square of the evaluation function repetitively using its unimodal. In the motor control, the evaluation function is the error between the reference value and measured value, and that is minimized by repetitive modification of FIR (Finite Impulse Response) filter. Using this method, the controller parameter, i.e. the filter coefficient is modified automatically. However, LMS algorithm takes time to calculate the minimum of square of the evaluation function [3]. This means that the adaptive control has a slow response performance.

In this paper, a switching control system to be compatible with the quick response performance of PID control and the automatic modification of the adaptive control is proposed. For that, the control input of PID control and that of the adaptive control need to be identical at the switching time. Therefore, the motor control algorithm to realize that is also presented.

### **2. Problem formulation**

In this section, PID control system, the adaptive control system and the proposed control system dealt with this paper is shown.

#### **2.1 PID control system**

In this subsection, PID control system is shown in the following figure:

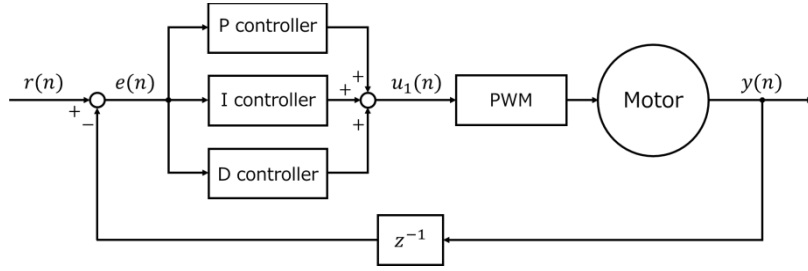


Fig. 1: PID control system

Here,  $r(n)$  is the reference speed,  $y(n)$  is the motor speed,  $e(n) (= r(n) - y(n - 1))$  is the error and  $n$  is the discrete-time.  $u_1(n)$  is the control input written by:

$$u_1(n) = K_p e(n) + K_i \sum_{i=0}^n e(i) + K_d \{e(n) - e(n - 1)\}. \quad (1)$$

Here,  $K_p$ ,  $K_i$ ,  $K_d$  are any real numbers and gains of P, I, D controller, respectively.

## 2.2 Adaptive control system using LMS algorithm

In this subsection, the adaptive control system is shown in the following figure:

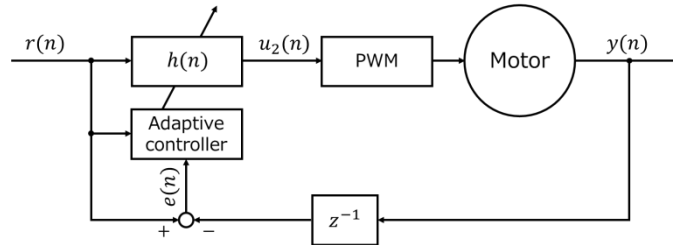


Fig. 2: Adaptive control system

Here,  $h(n)$  is the FIR (Finite Impulse Response) filter and written by  $h(n) = [h_0, h_1, \dots, h_N]$ , and  $N$  is the filter length. The control input  $u_2(n)$  is given by the convolution of the reference input  $r(n)$  and filter  $h(n)$  as:

$$u_2(n) = \sum_{i=0}^N h_i r(n - i). \quad (2)$$

In addition, the adaptive controller in Fig. 2 has a filter modification based on LMS method [3] as:

$$h(n + 1) = h(n) + \mu e(n) r(n). \quad (3)$$

Here,  $\mu$  is step size and any real number. Therefore, the adaptive control algorithm in Fig. 2 is summarized as follow:

- 1) Calculate the control input  $u_2(n)$  using Eq. (2).
- 2) Calculate the error  $e(n) (= r(n) - y(n - 1))$ .

- 3) Calculate the FIR filter  $h(n+1)$  using Eq. (3), and modify that filter.
- 4) Repetitive from 1) to 3).

### 2.3 Proposed control system

In this subsection, the proposed control system is shown in the following figure:

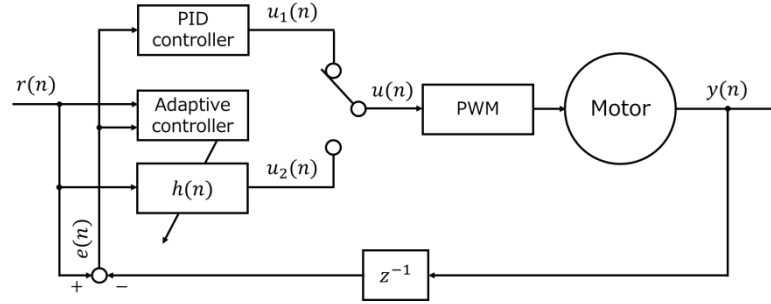


Fig. 3: Proposed control system

The characteristic of this control system is to calculate the output  $u_1(n)$  of PID controller and the output  $u_2(n)$  of the FIR filter in parallel, and to select either one as the resulting control input  $u(n)$  under the required condition. Therefore, it is enabled to obtain the quick response by selecting  $u_1(n)$  before the arrival for reference speed, and to obtain the self-modification by selecting  $u_2(n)$  after that. However, in order to switch from PID control to the adaptive control, control inputs  $u_1(n)$  and  $u_2(n)$  must be equal.

The purpose of this study is to present an algorithm to modify the FIR filter in Fig.3 using the control input  $u_1(n)$ .

## 3. Presented algorithms

### 3.1 Using the estimated impulse response of PID controller

In this subsection, an algorithm to estimate impulse response of PID controller and to modify the FIR filter as the estimated impulse response is presented. This algorithm is summarized as follow:

- 1) Calculate the error  $e_e(n) = \delta(n) - y_e(n-1)$ , here  $\delta(n) (= [1 \ 0 \ 0 \ \dots])$  is the impulse input and  $y_e(n)$  is the estimated motor speed.
- 2) Calculate the estimated impulse response of PID controller  $u_{1e}(n)$  as:

$$u_{1e}(n) = u_{1e}(n-1) + (K_p + K_i + K_d)e_e(n) + (K_p + 2K_d)e_e(n-1) + K_d e_e(n-2). \quad (4)$$

- 3) Modify the FIR filter  $h(n) = [h_0, h_1, \dots, h_N]$  as:

$$h_i = u_{1e}(n-i) \quad (i = 0, 1, \dots, N). \quad (5)$$

- 4) Calculate the estimated motor speed  $y_e(n)$  using the convolution of  $u_{1e}(n)$  in Eq. (4) and the identified motor model  $M(n)$  as:

$$y_e(n) = M(n) * u_{1e}(n). \quad (6)$$

5) Repetitive from 1) to 4) until switching from PID control to the adaptive control.

Here, the derivation process of above algorithm is shown. Using the difference method of PID controller, the control input  $u_1(n)$  is written by:

$$\begin{aligned} u_1(n) &= u_1(n-1) + K_p\{e(n) - e(n-1)\} + K_i e(n) \\ &\quad + K_d[\{e(n) - e(n-1)\} - \{e(n-1) - e(n-2)\}] \\ &= u_1(n-1) + (K_p + K_i + K_d)e(n) + (K_p + 2K_d)e(n-1) + K_d e(n-2). \end{aligned} \quad (7)$$

When the impulse input  $\delta(n) (= [1 \ 0 \ 0 \ \dots])$  is inputted as  $e(n) = \delta(n) - y_e(n-1)$ , the impulse response of PID controller can be estimated. PID controller can be thus expressed as FIR filter by the replacement from the estimated impulse response to FIR filter  $h(n) = [h_0, h_1, \dots, h_N]$ . Therefore, Eq. (4) and (5) are given.

In this way, the presented algorithm is obtained.

### 3.2 Using the measured control input of PID controller

In this subsection, an algorithm to modify the FIR filter using the control input of PID controller is presented. The presented algorithm in subsection 3.1 uses the estimated impulse response and the identified motor, then there is a possibility that the estimated/identified error is exist. Therefore, under the assumption that the reference input is constant, the following algorithm based on the measured value is summarized:

The reference input  $r(n)$  is assumed to be constant  $r$ .

- 1) Calculate the differential  $\Delta u_1(n)$  between the present control input  $u_1(n)$  and the previous control input  $u_1(n-1)$ .
- 2) Using the constant reference input  $r$ , modify the FIR filter as:

$$h_i = \frac{\Delta u_1(n-i)}{r} \quad (i = 0, 1, \dots, N). \quad (8)$$

- 3) Repetitive from 1) to 2) until switching from PID control to the adaptive control.

Here, the derivation process of above algorithm is shown. The reference input  $r(n)$  is assumed to be constant. In order for the control input of PID controller and that of adaptive controller are identical at  $n = 0$ :

$$h_0 r = u_1(0) \quad (9)$$

must be satisfied. From Eq. (9), the filter coefficient  $h_0$  is given by  $h_0 = u_1(0)/r (= \Delta u_1(0)/r)$ . Similarly, in order for the control input of PID controller and that of adaptive controller are identical at  $n = 1$ , using the differential  $\Delta u_1(n) = u_1(n) - u_1(n-1)$  and the constant reference input  $r$ :

$$h_1 r + h_0 r = u_1(1) = u_1(0) + \Delta u_1(1) \quad (9)$$

must be satisfied. From Eq. (9), in order to satisfy Eq. (10),  $h_0 = \Delta u_1(1)/r$  and  $h_1 = \Delta u_1(0)/r$ . Similarly, at  $n = 2$ :

$$h_2 r + h_1 r + h_0 r = u_1(2) = u_1(1) + \Delta u_1(2) \quad (11)$$

must be satisfied. Therefore, in order to satisfy Eq. (11),  $h_i = \Delta u_1(2 - i)/r$  ( $i = 0, 1, 2$ ). Thus, in order for the control input of PID controller and that of adaptive controller are identical at  $n = N$ , the filter coefficient  $h_i$  ( $i = 0, 1, \dots, N$ ) is modified by Eq. (8).

In this way, the presented algorithm is obtained.

#### 4. Numerical example

In this section, a numerical example using the presented algorithm in subsection 3.2 is shown in Fig. 4:

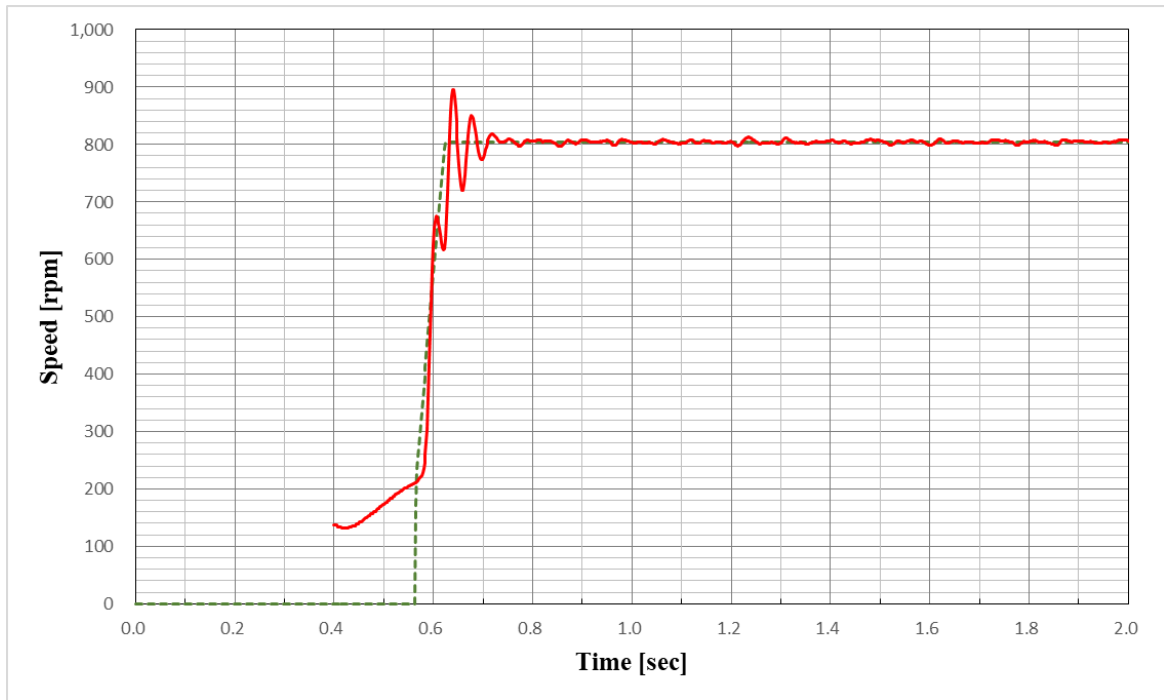


Fig. 4: Motor response using the presented algorithm in subsection 3.2

Here, the solid line is the motor speed  $y(n)$ , and the dotted line is the reference speed  $r(n)$ . The motor speed is measured from 0.4[sec] by back electromotive force. In addition, the control mode is the open-loop control from 0.0[sec] to 0.564[sec], PID control from 0.564[sec] to 0.690[sec] and the adaptive control from 0.690[sec]. Figure 4 shows that the control system is stable by PID control and the adaptive control. In addition, the enlarged view from 0.680[sec] to 0.700[sec] is shown in Fig. 5. Figure 5 shows that there is no negative effect for the motor speed response at the switching time. Therefore, control inputs of PID controller and the adaptive controller are identical.

In this way, we confirmed that the presented algorithm is surely switched from PID controller to the adaptive controller.

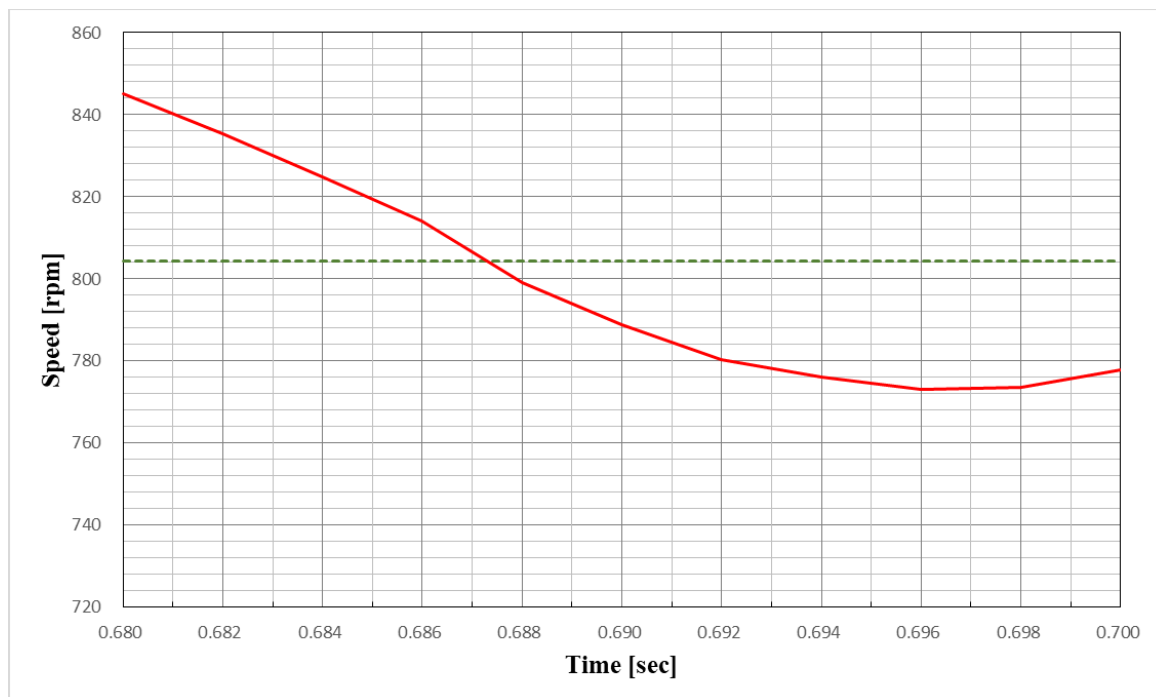


Fig. 5: Enlarged view of Fig. 4 from 0.680[sec] to 0.700[sec]

## 5. Conclusion

In this paper, the control system in Fig. 1 and two algorithms in subsection 3.1 and 3.2 are proposed in order to have advantages of PID control and the adaptive control. In addition, due to the motor control in practice, we found that PID control and the adaptive control are surely switched.

In the future, the proposed control system and algorithms are expected to expand into vector control system in order to compensate the motor torque. As a further application, the proposed control method is expected to apply for the mass-produced motor driver in order to modify the effect by the change of control environment automatically.

## References

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