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Invited Paper

Measurement of the centres of mass of test masses for free-fall interferometry

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Abstract. Free fall interferometers are employed, for example, in absolute gravimeters and gravity gradiometers. In the free-fall interferometers, rotation of falling test masses could cause one of the largest acceleration disturbances. This rotational acceleration disturbance of a test mass can be minimised by adjusting the centre of mass of the test mass with the optical centre of a retroreflector or a corner cube prism, which is embedded in the test mass. In this paper, a method of measuring the centres of mass of test masses using the weighbridge method developed for 1 kg standards is presented. The heights of the centres of mass of prototype test masses for a gravity gradiometer were determined with an uncertainty of ± 10 μm. Using the measured centres of mass, the magnitudes of the rotational acceleration disturbance were estimated to be less than 0.02 μGal. This level of disturbance is well below the target sensitivities of the absolute gravimeters and interferometric gravity gradiometers.

1. Introduction

Laser-interferometric gravity gradiometers are being developed for onboard measurements [1], observations of volcanic activities and field measurements [2]. In such a gravity gradiometer, differential acceleration of two test masses in free fall is measured by a laser interferometer. In these gravity gradiometers, rotation of the falling test masses could be one of the largest disturbances [2]. The rotational acceleration disturbance is proportional to the offset of the centre of mass from the optical centre of a test mass [2]. To estimate the magnitude of the rotational acceleration disturbance, the offset has to be measured with sufficient sensitivity. This paper briefly introduces a method of measuring the offset by applying the weighbridge method developed for 1 kg standards at the Bureau International des Poids et Mesures (BIPM) [3].

2. Experimental Method

The weighbridge method developed by R S Davis at BIPM [3] was applied for the measurement of the centres of mass of test masses for an interferometric gravity gradiometer. This method is capable of locating the centre of mass of a 1-kg test object to within a precision of several micrometers. A brief description of the operation principle and the experimental procedure is given below. A more detailed description of the experiment will be reported in a future paper.

2.1 Operation Principle

As shown in Fig. 1, the first and second measurements are carried out by setting the base of a test object against the left-hand wall of the weighbridge and the right-hand wall of the weighbridge, respectively. The first and second measurements give the changes in mass indicated by the balance m_1 and m_2, respectively. The balance used (METTLER TOLEDO, ME4002) has a capacity of 4.2 kg and a resolution of 0.01 g. The weighbridge was fabricated based on the details given in Fig. 2 of Ref. [3].

With the mass of the test object m_0 and the length of the span of the weighbridge L, the height of the centre of mass of the test object is given by the following equation [3]:

$$h=\left(\frac{m_0-m_1+m_2}{m_0}\right)\frac{L}{2}. \tag{1}$$

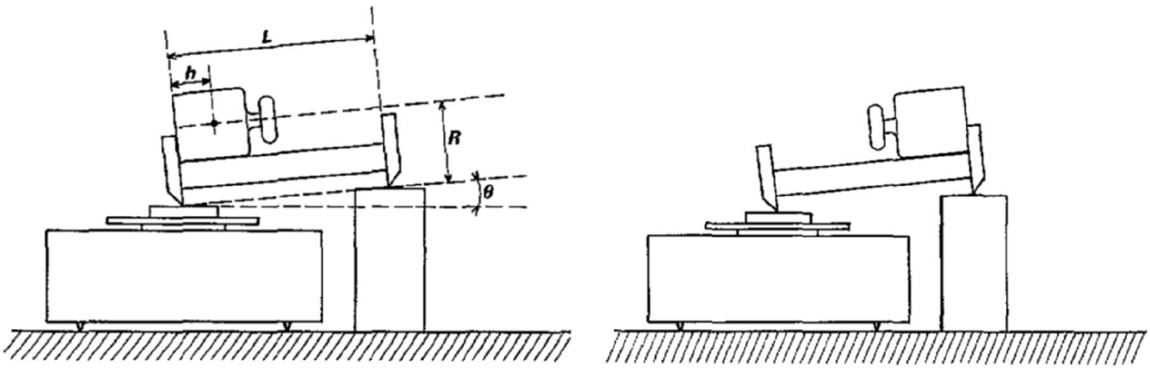

Fig. 1. A schematic view of the first measurement (left) and the second measurement (right) (quoted from Figs. 1 and 3 of Ref. [3]). A weighbridge is placed between the balance pan and a solid support. The changes in mass indicated by the balance m_1 and m_2 are obtained by the first measurement and the second measurement, respectively.
Code: h= height of the centre of mass, L= span of the weighbridge, R= distance between the centre of mass and the plane defined by the knives of the weighbridge, and θ = tilt angle.

2.2 Determination of the Span *L*

A brass cylinder with a height $2h$ of 60.017 ± 0.006 mm at 26.5 °C and a nominal diameter of 33.0 mm was used as a standard to determine L from Eq. (1). The first and second measurements of the standard were repeated five times within a day. The average value of L was 149.86 mm. The standard deviation was 0.017 mm. The systematic error due to the uncertainty in h of the standard propagates as $\delta L \sim \frac{L}{h}\delta h$ and was 0.030 mm in the experimental setup. Such a determination of L was made three times on different days in a month. The determined values of L were between 149.83 mm and 149.87 mm, and the average value was 149.85 mm. Vernier caliper measurements of L was 149.90 ± 0.05 mm and consistent with the determinations by the weighbridge method.

2.3 Tilt Effect

The efficiency of Eq. (1) in eliminating bias caused by levelling error was confirmed by performing measurements at different tilt angles of 3.3, 6.7, 10 and 13 mrad. The effect of the tilt in a determination of h was estimated to be less than 1 μm in the experimental setup and negligible.

2.4 Calibration

Another brass cylinder with the same dimensions as the standard was prepared and a slot of 3.0 mm wide and 3.0 mm deep was machined along a diameter at one end of the cylinder. The height of this cylinder was measured by a micrometer, $2h$ = 59.974 ± 0.006 mm at 26.5 °C. The height of the centre of mass of the cylinder with the slot was calculated by considering the measured dimensions and it was 30.151 ± 0.004 mm. The first and second measurements were carried out for this cylinder six times and the height of the centre of mass was estimated to be 30.14 ± 0.01 mm. The uncertainty indicates the root of the square sum of the standard error of 0.006 mm and the systematic error of 0.008 mm due to the uncertainty in L. The value by the weighbridge measurements was consistent with the theoretical value obtained by the calculation.

2.5 Prototype Test Masses

A pair of prototype test masses (A and B) were prepared for trial measurements. The prototype test mass A is embedded with a 10-mm beam-splitter cube and a corner cube prism with a diameter of 10 mm. The prototype test mass B is embedded with a corner cube prism with a diameter of 10 mm. These two test masses are to form an interferometer for measurements of gravity gradients.

3. Results and Discussion

The centres of mass of the prototype test masses were measured by the weighbridge method five times. The optical centres of the prototype test masses were measured by a micrometer. The measured values are listed in Table 1. The uncertainties in the centres of mass indicate the root of the square sum of the standard error and the systematic error in L. From these measurements, the offset of the centre of mass from the optical centre was 0.13 ± 0.01 mm for the prototype test mass A and 0.04 ± 0.01 mm for the prototype test mass B.

The rotational acceleration disturbance of a test mass is given by the following relation (e.g.[2]):

$$\boldsymbol{a_z = d \cdot \omega^2}, \tag{2}$$

where d is the offset and ω is the angular velocity of the test mass. The largest measured angular velocity was 1.0 ± 0.4 mrad/sec in a portable gravity gradiometer, being developed at Gunma University [2]. With this angular velocity and the estimated offsets, the magnitude of the rotational acceleration disturbance was estimated for the prototype test mass A and B. The results are shown in Table 1. The magnitudes of the rotational acceleration disturbance were sufficiently smaller than the target sensitivity of the portable gravity gradiometer, 0.1 μGal or 1×10^{-9} m/s^2. It should be noted that the standard deviation of the angular velocity, $\delta\omega$=0.4 mrad/sec, is propagated in the estimation of the uncertainties in the rotational acceleration disturbance.

Table 1. The results of the measurements. The centres of mass were determined by the weighbridge method. The optical centres were measured by a micrometer. The offsets are the differences between the centres of mass and the optical centres. Rotational acceleration disturbance was estimated using the angular velocity of 1.0 ± 0.4 mrad/sec observed in the portable gravity gradiometer [2].

	Centre of mass [mm]	Optical centre [mm]	Offset [mm]	Rotational acceleration disturbance [μGal]
Prototype test mass A	26.17 ± 0.01	26.04 ± 0.01	0.13 ± 0.01	0.013 ± 0.010
Prototype test mass B	14.40 ± 0.01	14.36 ± 0.01	0.04 ± 0.01	0.004 ± 0.003

4. Conclusion

The centres of mass of the prototype test masses were measured by applying the weighbridge method developed for kilogram standards at the BIPM. Using the estimated offsets of the centres of mass, the magnitudes of the rotational acceleration disturbance were estimated to be less than 0.02 μGal and sufficiently small for the portable gravity gradiometer being developed at Gunma University.

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