Frequency estimation method in Laser Doppler Interferometer using a Digitizer by sine fitting

Irfa Aji Prayogi^{1, a}, Akihiro Takita^{1, b} Mitra Djamal^{2, c} and Yusaku Fujii^{1, d} ¹Department of Mechanical Science and Technology, School of Science and Technology Gunma University 1-5-1 Tenjincho, Kiryu 376-8515, Japan

> ²Department of Physics, Faculty of Mathematics and Natural Science Institut Teknologi Bandung Jalan Ganesha 10, Bandung 40132, Indonesia

at13802281@gunma-u.ac.jp, btakita@gunma-u.ac.jp, cmitra@fi.itb.ac.id, dfujii@gunma-u.ac.jp

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Abstract. A method for estimating frequency from continuous digitized waveform using sine fitting was proposed. In this method, the frequency is estimated using sine fitting from the segment of continuous waveforms which divided by a constant period and a constant time. The performance of this method is evaluated by comparing with the zero crossing fitting method (ZFM), which is evaluating signals in numerical simulation. The result show that the sine fitting method has considerably lower error instead of the ZFM.

1. Introduction

Currently, the velocity measurement using a laser Doppler interferometer (LDI) is a common method since it has high precision and resolution [1-3]. The velocity is change corresponding to Doppler frequency shift, $f_{Doppler}$. In the LDI using two-frequency laser (f_1 , f_2), the velocity can be calculated as follow,

$$v = \left(\frac{\lambda}{2}\right) f_{Doppler},$$

$$f_{Doppler} = |f_b - f_r|,$$
 (1)

where the λ is the wavelength of the signal beam in the air, f_r is rest frequency which is the laser frequency difference $(f_r = |f_l - f_2|)$ and f_b is beat frequency which the frequency of beam signal modulated by Doppler frequency shift $(f_b = |f_l - f_2 + f_{Doppler}|)$. The Doppler frequency shift is the difference between beat frequency, f_b that changes proportionally to the velocity of the object and rest frequency, f_r , which is the reference signal. In the experiment, the f_b and f_r is measured separately using a frequency counter or a digitizer which is a high-speed analog to digital converter (ADC). The easiest way to measure the frequency in LDI is using a frequency counter. The resolution has been proven improved in the certain latest generation of the commercial frequency counter using multiple time stamp average continuous counting method [4]. However, it is not sufficient to measure frequency with sufficiently high resolution and sampling rate in this study, instead of frequency counter a highspeed digitizer is prefer used to record the whole wave profile. Then the frequency is estimated from the digitized waveform using computers.

This study is used to improve the frequency estimation in Levitation Mass Method (LMM) which is using a LDI to measure the velocity and acceleration. Various methods and algorithms have been proposed for estimating the frequency in LMM experiment such as zero crossing method (ZCM) was first suggested [5], zero crossing averaging method (ZAM) [6], zero crossing fitting method (ZFM)-period [7,8], constant gate time ZFM (CGT-ZFM) [9] and acceleration-constant zero crossing fitting method (AC-ZFM) [10]. In order to obtain a changes of frequency in time-window, the waveforms were divided into segments using a constant period [5-8,10] or a short constant time [9]

Currently, the ZFM was usually used for LMM experiment since it had a lower noise level compared to the other methods [5,6,9]. This method used a linear fitting to estimate the frequency and only used zero crossing data, which is obtained from the linear interpolation of two adjacent data near zero. It means not all waveform information was used in ZFM. In addition, this waste of data must cause the error of measurement. We think that the error can be improved if we utilize all waveform information by a nonlinear fitting method such as sine fitting [11]. This paper shows the performance of the sine fitting method which is compared with ZFM (period) for evaluating signal in the numerical simulation and processing data experiment of LMM.



2. Sine Fitting Method

Fig. 1. The method to estimate frequency using sine fitting (SFP using N-4 and SFT using constant-time, $ct=2 \mu s$)

In this study, the MATLAB curve fitting toolkit (sinefit.m) is used for fitting the waveform data. The curve fitting tool is used following fitting function:

$$y(t) = Asin(\omega t + \varphi), \tag{2}$$

where A is amplitude, ω is angular frequency, and φ is the phase. The fitting algorithm seeks the values of the A, ω , and φ , which is minimizing the square sum of differences data waveform, y and sine function model, y(t),

$$\sum_{i=0}^{N} [y_i - A_i \sin(\omega_i t + \varphi)]^2, \tag{3}$$

then the frequency is calculated as $\omega/2\pi$. There are two type of the proposed sine fitting method in this study i.e., the sine fitting period method (SFP) which is divide the output waveform into segment by a constant period. And the sine fitting constant-time method (SFT) which is divide the output waveform into segment by a short constant time. Figure 1 shows the difference of algorithm of SFP that is used 4-cycle period, N and SFT that is used a constant time, *ct* 2µs in frequency estimation.

3. Result and Discussion

3.1 Numerical Simulation

Here, the ideal signals were generated which represented a continuous back and forth motion as simulated signals. The equation 4 shows the simulation signals with the background noise of measurement, S_b and reference, S_r signals.

$$S_b = A \sin(2\pi f_r t + B \sin(2\pi f_d t) + \varphi_o) + dc + n(t),$$

$$S_r = A \sin(2\pi f_r t + \varphi_o) + dc + n(t),$$
(4)

where A, φ_{o} , and dc are the intensity, initial phase and direct current component of the simulation signals, respectively; B is the factor of the maximum Doppler shift; f_r is the frequency difference; f_d is the frequency of Doppler shift signal in signal beat; n(t) is the background noise which can be characterized as random Gaussian white noise. Table 1 shows the parameters value of simulation signals.

Table1. Parameter value of simulation signal

Parameter	A	fr	fd	φ_o	dc	В	
Value	2	1.5 x 10 ⁻⁶	100	π/7	5	1000	

The simulation signals are sampled to the digital waveform with the sampling rate of 20 MHz and the sampling number 5 M sample. The frequency is estimate every 100 periods in ZFM and SFP method. In order to make a comparable result with same temporal resolution, the SFT method is using a constant time, *ct* 0.065 ms. The NRMSD (Normal Mean Square Deviation) of acceleration is used to evaluate the estimation error of the methods which is calculated using Equation 5. It estimates the normalized square error between calculated and standard acceleration. The standard acceleration is calculated from signals without noise for each method.

$$NRMSD = \sqrt{\frac{n^{-1}\sum_{i=1}^{n} (a_s(i) - a(i))^2}{a_s max - a_s min}} \times 100\%,$$
(5)

where $a_s max$ and $a_s min$ is maximum and minimum standard acceleration, respectively. Figure 2 shows the errors of the acceleration in collision time. In the simulation, the maximum of Doppler shift is 0.1 MHz and the maximum acceleration is 20 ms⁻². When the SNR changes from 20 dB to 60 dB, the errors change from 1.20% to 0.012% of ZFM, from 0.55% to 0.005% of SFP and from 0.58% to 0.005% of SFT. Referring to paper [9], the error in acceleration for AC-ZFM changes from 0.8 % to 0.04 % in the same condition. And for CGT-ZFM, referring to paper [8], the error showed almost similar results with the ZFM. The results show that the sine fitting methods have a lower error than ZFM and AC-ZFM under the same condition.



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3.2 Estimation result using an experimental data

An experimental data of material testing with LMM [3] is used to evaluate the frequency estimation by proposed method. In this experiment, only one collision occurs. A Michelson type LDI is used to measure the velocity of collider. The Zeeman-type two wavelength He-Ne laser is used as light source which have a rest frequency approximately 1.76 MHz. A digitizer (NI PCI-5105, National Instruments Corp., USA) is used to recorded the signal from photodiodes (5 M sample each channel at a sampling rate 20 MHz with 8-bit resolution). The outputs of the measurement are digitized beat and rest waveform. Here, the frequency is estimated every 200 periods in ZFM and SFP. It has a time resolution is 10.63 ms which is used as constant time, *ct* in SFT to make a comparable evaluation.

Figure 3 shows plot of the Doppler shift frequency, $f_{Doppler}$, estimated by ZFM, SFP and SFT. The dividing a waveform by a constant time which applied in SFT caused the beat and rest frequency are coinciding. It results the noise level of $f_{Doppler}$ lower than the other method. The accuracy of acceleration is strongly depending on its $f_{Doppler}$ accuracy in the velocity calculation



Fig. 3. The Doppler frequency shift calculated using (a) ZFM (b) SFP (c) SFT

Figure 4 shows the acceleration calculated by (a) ZFM, (b) SFP, and (c) SFT. In this figure, it is proved that the SFT method can improve the resolution much without sacrificing the sampling rate in the acceleration calculation. The root mean square (RMS) of acceleration before and after collision time which representing the error of measurement is shown in Table 2. The relative error of measurement calculated using ZFM is 0.64 %, SFP is 0.61 %, and SFT is 0.05 % of maximum acceleration.



Fig. 4. Acceleration calculated using (a) ZFM (b) SFP (c) SFT

Method	Before [ms ⁻²]	After [ms ⁻²]
ZFM	0.155	0.136
SFP	0.148	0.130
SFT	0.010	0.011

Table2. RMS of acceleration before and after collision

4. Conclusion

From the numerical simulation results, it is demonstrated that the acceleration calculated by sine fitting method is more accurate than that by the zero-crossing methods. An acceleration which is calculated from experimental data using the SFT is evaluated considerably lower error than the ZFM-cycle and SFP in the comparable time resolution. The results showed that the sine fitting method especially SFT is superior to the zero-crossing methods.

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