Proceedings of International Conference on Technology and Social Science 2018 (ICTSS 2018)

Acoustic Finite Element Analysis Considering Air Viscosity in Narrow Rectangular-Cross Section

Manabu Sasajima[1, a], Takao Yamaguchi[2, b], Yue Hu[1, c]
and Mitsuharu Watanabe[1, d]

[1]Foster Electric Co., Ltd., 1-1-109 Tsutsujigaoka, Akishima, Tokyo, 196-8550 Japan

[2]Gunma University, 1-5-1 Tenjin-cho, Kiryu, Gunma, 376-8515 Japan

[a]<sasajima@foster.co.jp >, [b]<yamagme3@gunma-u.ac.jp >, [c]<fhuy@foster.co.jp >,

[d]<mtwatanabe@foster.co.jp>

Keywords: simulation, FEM, viscosity, acoustic, propagation

Abstract. This study deals with the propagation of sound waves through air in narrow rectangular cross-sectional pathways. In such pathways, the speed of sound and the phase of sound waves are affected by viscous air damping. We have developed a new finite element method (FEM) that considers the effects of air viscosity. This method was developed as an extension of the existing FEM for porous sound-absorbing materials. The results of a numerical calculation for a narrow three-dimensional rectangular cross-sectional tube model were validated using the proposed FEM. This validation was carried out through comparison with results obtained from existing calculation methods. Furthermore, the relative error between the proposed method and the theoretical method was also validated.

1. Introduction

The viscosity of air in narrow pathways causes viscous damping. As a result of this, the speed of sound decreases and phase delay occurs. Therefore, the effect of air viscosity must be considered in order to carry out accurate acoustic analyses for small electronic devices such as intra-concha and insert earphones. This effect is usually not considered in conventional acoustic analysis. In this study, we developed a new FEM that considers the effects of air viscosity in narrow portions of a sound pathway inside small electronic devices.

This method was developed as an extension of the acoustic FEM proposed by Yamaguchi(1), (2) for a porous sound-absorbing material. We carried out a numerical analysis in the frequency domain using our acoustic solver, which utilizes the proposed FEM(3), (4). For the numerical calculations, we used a tube model with a rectangular cross section. Then, we compared the results obtained from the proposed FEM with those obtained through theoretical analysis and through the generally used finite element analysis, which does not consider the effects of air viscosity.

2. Numerical Procedures

We have developed a new FEM that incorporates air viscosity at small amplitudes. Figure 1 shows the direct Cartesian coordinate system with a constant strain element for a three-dimensional (3D) tetrahedron. Here, u_x, u_y, and u_z are the displacements in the x, y, and z directions, respectively, at arbitrary points in the element. The strain energy $\widetilde{U}$ can be expressed as follows:

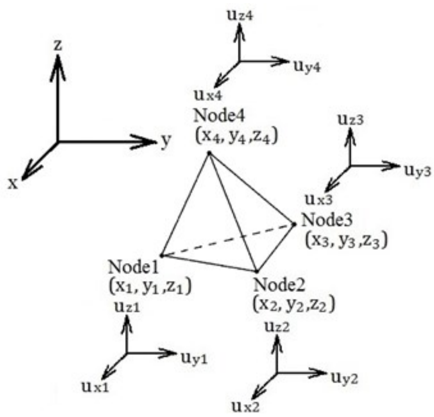

Fig. 1 Direct Cartesian coordinate system with a constant strain element.

$$\widetilde{U}=\frac{1}{2}E\iiint_e\left(\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}+\frac{\partial u_z}{\partial z}\right)^2 dxdydz \tag{1}$$

where E is the bulk modulus of elasticity of air. The time derivative of particle displacement is denoted by $\dot{u}$. Therefore, the kinetic energy $\widetilde{T}$ can be expressed as follows:

$$\widetilde{T}=\frac{1}{2}\iiint_e \rho\{\dot{u}\}^T\{\dot{u}\}dxdydz \tag{2}$$

where ρ is the effective density of the element, and T represents a transposition. The viscous energy $\widetilde{D}$ of a viscous fluid can be expressed as follows:

$$\widetilde{D}=\iiint_e \frac{1}{2}\{\overline{T}\}^T\{\Gamma\}dxdydz \tag{3}$$

where $\{\overline{T}\}$ is the stress vector attributed to the viscosity. The relationship between the particle velocity and the stress can be expressed as follows:

$$\{\overline{T}\}\equiv\begin{Bmatrix}\tau_{xx}\\ \tau_{yy}\\ \tau_{zz}\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{zx}\end{Bmatrix}=\begin{bmatrix}\frac{4}{3}\mu\frac{\partial}{\partial x} & -\frac{2}{3}\mu\frac{\partial}{\partial y} & -\frac{2}{3}\mu\frac{\partial}{\partial z}\\ -\frac{2}{3}\mu\frac{\partial}{\partial x} & \frac{4}{3}\mu\frac{\partial}{\partial y} & -\frac{2}{3}\mu\frac{\partial}{\partial z}\\ -\frac{2}{3}\mu\frac{\partial}{\partial x} & -\frac{2}{3}\mu\frac{\partial}{\partial y} & \frac{4}{3}\mu\frac{\partial}{\partial z}\\ \mu\frac{\partial}{\partial y} & \mu\frac{\partial}{\partial x} & 0\\ 0 & \mu\frac{\partial}{\partial z} & \mu\frac{\partial}{\partial y}\\ \mu\frac{\partial}{\partial z} & 0 & \mu\frac{\partial}{\partial x}\end{bmatrix}\begin{Bmatrix}\dot{u}_x\\ \dot{u}_y\\ \dot{u}_z\end{Bmatrix} \tag{4}$$

where $\dot{u}_x$, $\dot{u}_y$, and $\dot{u}_z$ are the particle velocities in the x, y, and z directions, respectively, at arbitrary points in the element, and μ is the coefficient of viscosity of air. In the above equation, $\{\Gamma\}$ is the strain vector. The relationship between the particle velocity and the strain can be expressed using the constant strain element of a 3D tetrahedron, as shown in Figure 2.

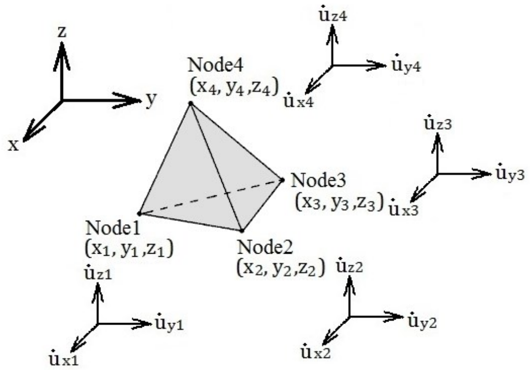

Fig. 2 Relationship between the particle velocity and the strain.

Next, we consider the formulation of the motion equation of an element for the acoustic analysis model that considers viscous damping. The potential energy $\widetilde{V}$ can be expressed as follows:

$$\widetilde{V} = \int_{\Gamma} \{u\}^T \{\overline{P}\} d\Gamma + \iiint_e \{u\}^T \{F\} dxdydz \tag{5}$$

where $\{\overline{P}\}$ is the surface force vector, $\{F\}$ is the body force vector, and $\int_{\Gamma} d\Gamma$ represents the integral of the element boundary. The total energy $\widetilde{E}$ can be derived using the following expression:

$$\widetilde{E} = \widetilde{U} + \widetilde{D} - \widetilde{T} - \widetilde{V} \tag{6}$$

We can obtain the following discretized equation of an element using the Lagrange equation:

$$\frac{d}{dt}\frac{\partial \widetilde{T}}{\partial \dot{u}_{ei}} - \frac{\partial \widetilde{T}}{\partial u_{ei}} + \frac{\partial \widetilde{U}}{\partial u_{ei}} - \frac{\partial \widetilde{V}}{\partial u_{ei}} + \frac{\partial \widetilde{D}}{\partial u_{ei}} = 0 \tag{7}$$

where u_{ei} is the i^{th} component of the nodal displacement vector $\{u_e\}$, and $\dot{u}_{ei}$ is the i^{th} component of the nodal particle velocity vector. We obtain the following discretized equation of an element by substituting (1)–(5) into (7):

$$-\omega^2 [M_e]\{u_e\} + [K_e]\{u_e\} + j\omega [C_e]\{u_e\} = \{f_e\} \tag{8}$$

We use $\{\dot{u}_e\} = j\,\omega\,\{u_e\}$ in this equation because a periodic motion with an angular frequency, ω, is assumed. $[M_e]$, $[K_e]$, $[C_e]$, and $\{f_e\}$ represent the element mass matrix, element stiffness matrix, element viscosity matrix and nodal force vector, respectively.

3. Calculation

3.1 Damping analysis using the 3D FEM

To verify our method, we carried out an acoustic damping analysis of tubes using a 3D FEM. As shown in Figure 3, the model was a 1/4 solid that was symmetrical about the *x-z* and *y-z* planes. The width of the model was 0.25 mm, the height was 0.25 mm, and the length was 16.60 mm. The model utilized 3D tetrahedral elements with four nodes. There were 25 divided elements along the length (*z* direction), and 10 layers along both the height (*y* direction) and width (*x* direction). Both ends of the model were closed. The effective density was ρ_0 = 1.2 kg/m^3, the coefficient of viscosity was μ = 1.82 × 10^{-5} Ns/m^2, the elasticity of the real part of the complex volume was E_0 = 1.4 × 10^5 Pa, and the speed of sound was c = 340 m/s (in air). For the boundary conditions, the particle displacements of all the outside nodes in contact with the surfaces were fixed, except in the planes of symmetry. Figure 4 shows the contour of the particle displacements and the isosurface view of the model obtained using the proposed FEM close to the resonance conditions of 10,200 and 20,400 Hz. As can be seen, the magnitudes of the displacements of the particles change significantly near the contact surface. However, the displacements become flatter with increasing distance from the contact surface.

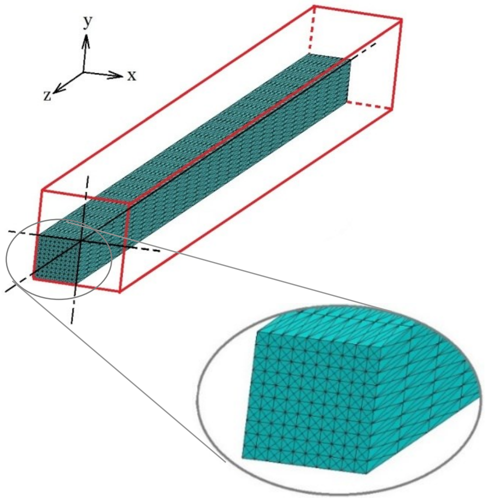

Fig. 3 rectangular cross-sectional model.

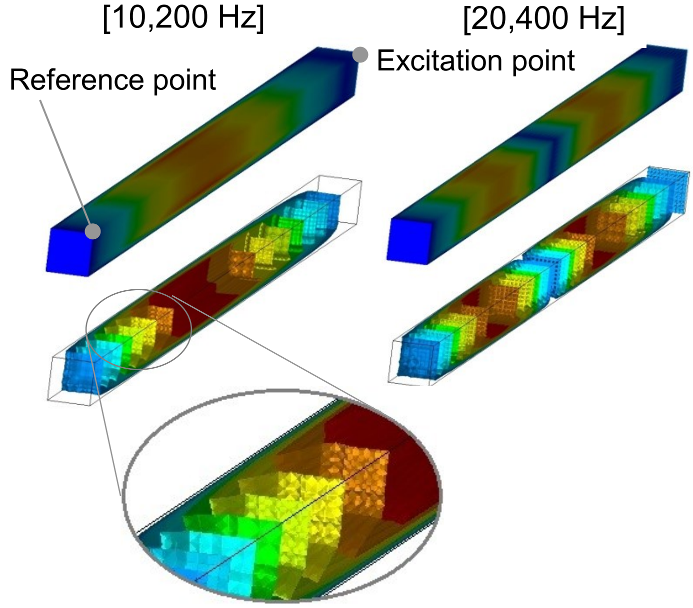

Fig. 4 Contour of the distribution of the particle displacement and isosurface view.

3.2 Theoretical damping analysis

We carried out a theoretical analysis of the resonant response of the rectangular cross-sectional pathway to verify the proposed FEM. The frequency response of the pressure may generally be calculated using the following equation(5):

$$P = -j\rho c v_0 e^{j\omega t} \frac{\cos k(x-l)}{\sin kl} \tag{9}$$

where ρ is the density of air, c is the speed of sound, l is the length of the tube, x is the position of a reference point, $k = \omega/c$, v_0 is the excitation velocity, and t is the time. In this equation, we introduce the complex speed of sound, c*, and the complex effective density, ρ_c*, to take into account the attenuation of air due to viscosity. We have replaced the speed of sound and the density with the complex speed of sound and the complex effective density, respectively, as shown below in (10).

$$\rho \Rightarrow \rho_c^*, \quad c \Rightarrow c^* \tag{10}$$

Substituting (10) into (9) and assuming the width of the tube model to be infinite, as shown in Figure 5, the effective density may be expressed as follows(6):

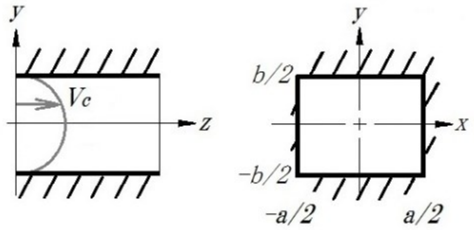

Fig. 5 Tube model of a rectangular section and velocity, *Vc*.

$$\rho_c^* = \frac{\rho_0}{F(\eta)} \tag{11}$$

where ρ_0 is the mass density. $F(\eta)$ and c^* are defined as follows:

$$F(\eta) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\frac{4j\omega}{\eta a^2 b^2}\frac{1}{\alpha_i^2\beta_j^2\left(\alpha_i^2+\beta_j^2+\dfrac{j\omega}{\eta}\right)} \qquad \alpha_i = \left(i+\frac{1}{2}\right)\frac{\pi}{a}, \beta_j = \left(j+\frac{1}{2}\right)\frac{\pi}{b} \tag{12}$$

$$c^* = \sqrt{\frac{\kappa}{\rho_c^*}} = \sqrt{\frac{\gamma p_0}{\rho_c^*}} \tag{13}$$

where a and b are the distances between the contact surfaces in the x and y directions, respectively; κ is the bulk modulus; p_0 is the atmospheric pressure; η is the coefficient of viscosity; and γ is the specific heat for a constant volume. The complex density and the complex sound velocity for the case of $a = b = 0.5$ mm are shown in Figures 6 and 7. By calculating the frequency response using (9) and the values of these parameters, the theoretical solution that considers the viscosity is obtained.

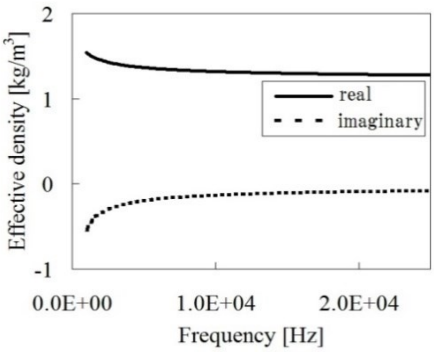

Fig. 6 Effective density ρ_c* of the model.

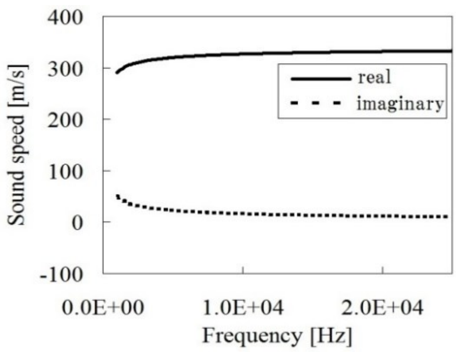

Fig. 7 Speed of sound *c** in the model.

3.3 Verification and comparison of the proposed method

We analyzed the frequency responses using the proposed FEM and compared the results with those from the previously described theoretical method, which considers the viscosity, and with those from the conventional FEM, which does not consider the attenuation. Figure 8 compares the analytical results for models in which a = b = 0.5 mm. The condition for excitation was constant displacement excitation. From Figure 8, we determined the effect of damping on the results obtained from the proposed FEM, and on the results obtained from the theoretical method. The results obtained using the conventional FEM do not exhibit attenuation of the resonance peaks. In addition, it may be observed that the results obtained using the proposed FEM and the theoretical method are approximately the same over the entire frequency domain.

4. Conclusion

We have developed a new acoustic FEM that considers the damping effects of air viscosity. We compared the sound pressure versus frequency characteristics obtained using the proposed method with those obtained using the theoretical method and the conventional acoustic FEM, which does not consider the effects of air viscosity in narrow rectangular-cross section pathway models. The comparison showed that the obtained results are in good agreement. Therefore, the proposed acoustic FEM was confirmed to have a good analytical accuracy. It was also confirmed that the numerical results of the proposed method converged towards the theoretical solution. It was therefore possible to confirm the effectiveness of this approach.

References

[1] T. Yamaguchi, J. Tsugawa, H. Enomoto and Y. Kurosawa, " Layout of Sound Absorbing Materials in 3D Rooms Using Damping Contributions with Eigenvectors as Weight Coefficients", ***Journal of System Dynamics***, Vol.4-1:166-176, 2014.

[2] T. Yamaguchi, Y. Kurosawa and H. Enomoto, " Damping Vibration Analysis Using Finite Element Method with Approximated Model Damping for Automotive Double Walls with a Porous Material", ***Journal of Sound and Vibration***, 325:436-450, 2009.

[3] M. Sasajima, T. Yamaguchi and A. Hara, " Acoustic Analysis Using Finite Element Method Considering Effects of Damping Caused by Air Viscosity in Audio Equipment", ***Applied Mechanics and Materials***, Volume 36, 2010, pp.282–286

[4] M. Sasajima, M. Watanabe, T. Yamaguchi, Y. Kurosawa, and Y. Koike, " Acoustic Analysis with Consideration of Damping Effects of Air Viscosity in Sound Pathway", ***World Academy of Science, Engineering and Technology***, Issue 78, 2013, pp.1011–1016

[5] J.W. Strutt, B. Rayleigh, ***The Theory of Sound (Second revised and enlarged edition)***, New Yoke Dover Publications, Inc., 1945.

[6] J.F. Allard, N. Atalla, ***Propagation of sound in Porous Media(Second Edition)***, A John Wiley and Sons, Ltd., Publication, 2009.