A Design Method of Sliding Mode Controller for EPS System Combined With Vehicle Motion

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Abstract. Electric power steering (EPS) is gradually replacing the conventional hydraulic power steering system in modern cars. The main advantages of EPS system are the ability to minimize energy consumption and its applicability to autonomous driving. In order to achieve precise estimation and control in EPS systems combined with vehicle motion, sliding mode observer (SMO) and sliding mode control (SMC) are introduced. SMO and SMC can explicitly consider disturbance and parameter variation in its design. Designed SMO and SMC are compared and evaluated through simulations.

1. Introduction

Considering all actuators of modern cars, the steering system is the most important one to design the feedback of the vehicle manoeuvrability. Electric Power Steering (EPS) is intended to assist the human steering by an electric motor. EPS is gradually replacing the conventional hydraulic power steering system in modern cars because of its good fuel efficiency and its applicability to autonomous driving [1]. From the point of view of the demand in recent years, EPS market is expanding rapidly.

EPS system is basically composed of the steering wheel, the electronic control unit, steering assist motor, angular sensor and its torque sensor. The EPS system uses steering assist motor to assist power by measuring and analyzing the direction and angle of steering wheel, and it potentially has applicability to autonomous driving. Therefore the control of the steering assist motor is core technology not only for the EPS system but also driving [2-4].

It is well known that, sliding mode control is a powerful control method with strong robustness to handle system's parametric uncertainties and external disturbances with bounded uncertainty information. In comparison with other robust control approaches, sliding mode control has the advantage [5,6].

In this research, a control system that integrates EPS system and vehicle's motion is firstly constructed. By introducing sliding mode observer (SMO: Sliding Mode Observer) to the constructed control object, improvement of the estimation accuracy against disturbance and parameter variation is achieved. In addition, it is shown that the sliding mode controller (SMC: Sliding Mode Controller) allows steering system not to be affected by disturbance and parameter variation compare to conventional controller. The effectiveness of the proposed SMO and SMC approach is verified in simulations by comparing with the conventional linear quadratic control method.

2. Modeling of EPS and Vehicle Systems

The adopted EPS model in this research is show in Fig. 1. The model is combined with the linear vehicle dynamics shown in Fig. 2. Parameters and variables used in Figs. 1 and 2 are shown in Table 1 and 2, respectively. The equation of motion of the steering system is expressed by Eq. (1) and Eq. (2), and the equation of motion of the vehicle is represented by the Eq. (3) and Eq. (4).

$$\left(I_H \dot{\theta}_H + K_H (\theta_H - \theta_M) = T_H\right) \tag{1}$$

$$\left(I_M \ddot{\theta_M} + C_T \dot{\theta_M} + K_T \theta_M + K_H (\theta_H - \theta_M) = T_M\right)$$
(2)

$$\left\{ mV + \frac{2(L_f K_f - L_r K_r)}{V} \right\} \gamma + mV\dot{\beta} + 2(K_f + K_r)\beta = 2K_r\sigma + F_G$$
(3)

$$\left(I_{\nu}\dot{\gamma} + \frac{2(L_{f}^{2}K_{f} + L_{r}^{2}K_{r})}{V}\gamma + 2(L_{f}K_{f} - L_{r}K_{r})\beta = 2L_{f}K_{f}\delta\right)$$
(4)

Output of the lateral acceleration sensor is expressed by Eq. (5).

$$G_{\gamma} = \dot{v_{\gamma}} + V\gamma \tag{5}$$

Table 1 Parameters for EPS model

I_h	0.0071 kgm ²	Steering wheel inertia
I_m	0.0094 kgm ²	Motor inertia
I_T	3.0 kgm ²	Tire inertia
K _h	135 Nm/rad	Torsion bar stiffness
C_T	389 Nm/rad	Tire damping
N_T	16.2	Steering gear ratio
N _m	18	Motor gear ratio
m	1188 kg	Vehicle mass
V	100 km/h	Vehicle velocity
I_{v}	1610 kgm ²	Vehicle yawing inertia
L_f	0.919 m	Front Axle - C.G. distance
L_r	1.471 m	Rear Axle - C.G. distance
L_w	0.4 m	Cross wind - C.G. distance
K_f	57153 N/rad	Front cornering power
K _r	78139 N/rad	Rear cornering power
ζ	0.0546 m	Trail

Table 2 Variables in EPS model

θ_h	[rad]	Steering wheel angle
θ_m	[rad]	Motor angle
$ heta_T$	[rad]	Tire angle
γ	[rad/s]	Yaw rate
β	[rad]	Vehicle slip side angle
T_h	[Nm]	Steering torque
\overline{T}_{M}	[Nm]	Motor torque
F_{w}	[Nm]	cross wind







Fig. 2. Vehicle model.

In these equations, θ_H , δ , I_H , I_M and K_H are the steering wheel angle, the motor angle (tire steering angle), the steering wheel inertia moment, the motor inertia moment, and the torsion bar stiffness, respectively. These amounts are expressed converting on tire axes. As a result, the following relationships are held

$$\theta_H = \frac{\theta_h}{N_t}, \ \delta = \frac{\theta_m}{N_t N_m}, \ I_H = {N_t}^2 I_h \tag{6}$$

$$T_{H} = N_{t}T_{h}, I_{M} = (N_{t}N_{m})^{2}I_{m} + I_{T}, K_{H} = N_{t}^{2}K_{h}$$
(7)

where N_t and N_m are the gear ratios of the steering and the motor to the tire axis, respectively. From these equations, the integrated state equation is represented as

$$\dot{x}(t) = Ax(t) + Bu(t) + E_1 T_H + E_2 F_w y(t) = Cx(t) \quad u(t) = T_M$$
(8)

where the state vector x and the output vector y are

$$\begin{aligned} x &= [\beta, \gamma, \theta_H, \dot{\theta}_H, \delta, \dot{\delta}]^T, \\ y &= [G_y, \gamma, \theta_H, \dot{\theta}_H, \delta, \dot{\delta}]^T \end{aligned}$$

The state matrix A, the input matrix B and the output matrix C are modified from Eq.(1)-(5).

3. Design of SMC and SMO

3.1 Application of Sliding Mode Observer to EPS system

Fig.3 shows the block diagram of SMO. If there are variations or disturbances in the system parameters, the linear observer is not sufficient in the control design due to the estimation errors. In this section, taking disturbances and parameter variations of the EPS system into consideration, the sliding mode observer (SMO) which can provide robust and accurate estimation is proposed. The SMO is represented by

$$\begin{aligned} \dot{x}(t) &= A_{SMO} \hat{x}(t) + B_{SMO} \left(u_{eq}(t) + M(x, y, \rho_2) \right) \\ \hat{y}(t) &= C_{SMO} \hat{x}(t) \\ u_{eq}(t) &= L \left(y(t) - \hat{y}(t) \right) \\ M(x, y, \rho_2) &= \begin{cases} -\frac{FCe}{||FCe||} \rho_2(t, V) & S(e) \neq 0 \\ 0 & S(e) = 0 \end{cases} \\ \rho_2(t, V) &= k_2 + f_2(x, V) \quad (k_2 \text{ is positive constant}) . \end{aligned}$$

 $u_{eq}(t)$ represents linear input and $M(x, y, \rho_2)$ represents nonlinear input. $\rho_2(t, V)$ is a nonlinear gain to adjust the compensation effect for the nonlinear factors. The variable parameters assumed in this system are velocity and moment of inertia. Disturbances are steering torque and crosswind to a vehicle. In order to compensate for these nonlinear terms, the nonlinear part $M(x, y, \rho_2)$ is divided into constant part and variable part which change according to state variable and velocity. This prevents the nonlinear gain from unnecessarily increasing when there is no velocity variation. As a result, it is possible to ensure stability with no chattering, and to improve the estimation accuracy of state variables.

3.2 Design of Disturbance Estimation

In order to estimate input disturbance, an inverse system using estimated state variables and system parameters is constructed. Eq. (9) shows a generalized model of Eq. (8) for considering unknown disturbance. Here, $u_1(t)$ is known input and $u_2(t)$ is unknown disturbance.

$$\dot{x}(t) = Ax(t) + Bu_1(t) + Eu_2(t) y(t) = Cx(t)$$
(9)

Eq. (9) is modified,

$$Eu_2(t) = \dot{x}(t) - Ax(t) - Bu_1(t)$$

Then, $\dot{x}(t) = C^{-1}\dot{y}(t)$ is substituted as follows
 $Eu_2(t) = C^{-1}\dot{y}(t) - Ax(t) - Bu_1(t)$

Taking $x(t) \rightarrow \hat{x}(t), u_2(t)$ is derived

$$u_2(t) = E^{-1}(C^{-1}\dot{y}(t) - A\hat{x}(t) - Bu_1(t))$$

From the above equation, the unknown disturbance $u_2(t)$ can be estimated.

3.3 Design of Sliding Mode Controller to EPS system

In this section, sliding mode control is applied to the steering system combined with vehicle motion. First, the switching function is set as follows.

$$\sigma = Sx$$

The equivalent control input is described as follows,

$$u_{eq} = -(SB)^{-1}SAx$$

In order to generate the sliding mode on the switching surface, the control input is defined as

$$\mathbf{u} = u_{eq} - \rho_1(x, t) \frac{\sigma}{\|\sigma\|} \tag{10}$$

$$p_1(t,V) = k_1 + f_1(x,V) + |\hat{F}_w| \quad (k_1 \text{ is positive constant})$$
(11)

The first term on the right side of Eq. (10) is the equivalent control input, and the second term is the switching function for generating the sliding mode. In the nonlinear gain $\rho_1(t, V)$, $f_1(x, V)$ in Eq. (11) is set as a function of velocity V, and the absolute value of the estimated crosswind disturbance is added. This compensates for upper value fluctuation of parameter change and disturbance. In this way, by changing the nonlinear gain $\rho_1(t, V)$ with respect to disturbances and parameter variations such as velocity and self-aligning torque, stability is assumed without causing chattering by nonlinear gain, and deterioration of the control performance is remarkably improved.

The control system integrating with the above controllers is shown in Fig. (4). By adding the disturbance estimated by SMO to the nonlinear gain of the sliding mode controller, the control system ensure the stability to the disturbance.



4. Simulations

4.1 Evaluation of step response to velocity variation

In order to evaluate the control system designed in the previous section, the simulation is performed in case of the step input of motor torque with parameter variation due to velocity change using the theoretical model shown in Eq. (8). The conventional method utilized the control input of Eq. (12), and the linear quadratic gains were determined by trial and error so that the rising time of proposed and conventional are equal.

$$u = K_1 \theta_H + K_2 \dot{\theta}_H + K_3 \delta + K_4 \dot{\delta}$$
(12)

Figs. 5 and 6 show the time responses when the step signal is applied to the motor torque. In simulation, the vehicle variation was varied from the nominal model V = 40 km/h to V = 20 km/h and V = 60 km/h. Fig. 5 shows time response of the steering angle of the proposed method, and Fig. 6 shows that of the conventional method. In contrast to the conventional method, it is seen that the proposed method has less perturbation in time response due to variations in vehicle velocity. That is, the proposed method shows that unintended deterioration of the steer feel can be suppressed even if the parameter fluctuates.

4.2 Evaluation to crosswind disturbance

In addition to the step input of motor torque in the previous section, the simulation results when adding crosswind disturbance to the vehicle are evaluated in Figs. 7 and 8. In simulation, the vehicle velocity is varied not only for the nominal model V = 40 km/h to V = 20 km/h and V = 60 km/h.

Fig. 7 shows the time response of the yaw rate of the proposed method, and Fig. 8 shows that of the conventional method. As in the previous section, deterioration of stability due to velocity fluctuation is suppressed in the yaw rate, and it is noticed that the proposed system is not affected by crosswind disturbance as compared with the conventional method.



Fig. 5. Step responses with parameter variation (proposed).



Fig. 6. Step responses with parameter variation (conventional).



5. Conclusion

In this research, a control system that integrates EPS system and vehicle's motion was firstly constructed. By introducing sliding mode observer and sliding mode controller to the constructed control object, proposed controller allows steering system not to be affected by disturbance and parameter variation compare to conventional controller. The effectiveness of the proposed design approach was confirmed by simulation compared with the conventional method. Future works will be the experimental verification.

References

- S. Endo, "EPS control design method for steering mechanism characteristic compensation to reduce the efforts required for handling", *Transactions of the JSME*, Vol. 82, No. 844, pp.1-18, 2016.
- [2] S. Takehara, and T. Yoshioka, "Improvement of Steering and Vehicle Characteristics due to Electric Power Assist Steering with Disturbance Observe", *Transactions of the JSME*, Vol. 70, No. 835, 2004.
- [3] D. Watanabe, M. Kataoka, T. Fujii, and M. Iwase, "Development of Electric Power Steering Control based on Alignment Torque", *DENSO TECHNICAL REVIEW*, Vol. 15, pp. 69-71, 2010.
- [4] M.K.Hassan, N.A.M.Azubir, H.M.I.Nizam, S.F.Toha, B.S.K.K.Ibrahim, "Optimal Design of Electric Power Assisted Steering System (EPAS) Using GA-PID Method", *International Symposium on Robotics and Intelligent Sensors*, Vol.41, pp.614-621, 2012.
- [5] H. Du, Z. Man, J. Zheng, A. Cricenti, H. Wang, and Y. Zhao, "A Novel Sliding Mode Control for Lane Keeping in Road Vehicles", *Proceedings of the 2016 International Conference on Advanced Mechatronic Systems*, pp. 289-294, 2016.
- [6] Zhe, Jinchuan, Zhihong, "Advanced control design for a vehicle steer-by-wire system by using adaptive fast nonsingular terminal sliding mode", *Proceedings of the 2016 International Conference on Advanced Mechatronic Systems*, pp.212-217, 2016.