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Impedance Expansion Method —A Circuit Modeling Technique for Electrically-Very-Small Wireless Systems—

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Abstract. The author has proposed a circuit-modeling technique for electrically-very-small wireless systems called the impedance expansion method (IEM). The IEM is based on expanding the self-/mutual impedances in the method of moments into the Laurent series with respect to the complex angular frequency. This paper describes the basic concept of the IEM, and then shows an example application to a wireless power transfer system.

1. Introduction

Electrically-very-small wireless systems are widely used. Typical examples include electrodes for intrabody communications [1], coils for wireless power transfer (WPT) systems [2], etc. It is well known that undesired resonances or radiations may occur at the usual operating frequencies of these devices, and they affect their operating characteristics or cause noises [3]. Besides, the electromagnetic characteristics of electrically-very-small devices may be approximated by equivalent circuits. This approach has the benefits such that:

1. small-scale circuit models can easily be analyzed via theoretical approaches, which give us an insight on the operation mechanism of the devices;
2. the interaction between the electromagnetic fields and the non-linear electronic circuits can be analyzed only by importing the equivalent-circuit parameters into versatile circuit simulators.

Recently, the author has proposed a circuit-modeling technique for electrically-very-small wireless systems called the impedance expansion method (IEM) [4]–[8]. The IEM is based on expanding the self-/mutual impedances in the method of moments (MoM) [9] into the Laurent series with respect to the complex angular frequency. This paper describes the basic concept of the IEM, and then shows an example application to a wireless power transfer system.

2. Basic Concept

In the MoM, the current distributions $\boldsymbol{J}(\boldsymbol{r})$ are expanded as follows:

$$\boldsymbol{J}(\boldsymbol{r}) = \sum_{n=1}^{N} I_n \boldsymbol{F}_n(\boldsymbol{r}),$$

where $\boldsymbol{r}$ is the observation point, I_n is the nth current coefficient, $\boldsymbol{F}_n(\boldsymbol{r})$ is the nth basis function, and N is the number of basis functions. The current coefficients can be obtained by solving the following system of linear equations.

$$\sum_{n=1}^{N} Z_{mn} I_n = V_m, \quad m = 1, \dots, N, \tag{1}$$

where V_m is the mth voltage coefficient, and Z_{mn} is the self-/mutual impedance between basis functions $\boldsymbol{F}_m(\boldsymbol{r})$ and $\boldsymbol{F}_n(\boldsymbol{r})$, which is expressed as follows:

$$\begin{aligned} Z_{mn} = s\frac{\zeta}{4\pi c}\int_S \int_S \boldsymbol{F}_m(\boldsymbol{r}) \cdot \boldsymbol{F}_n(\boldsymbol{r}')\frac{e^{-sR/c}}{R}dS'dS \\ +\frac{1}{s}\frac{\zeta}{4\pi c}\int_S \int_S [\nabla \cdot \boldsymbol{F}_m(\boldsymbol{r})] \cdot [\nabla' \cdot \boldsymbol{F}_n(\boldsymbol{r}')]\frac{e^{-sR/c}}{R}dS'dS, \end{aligned} \tag{2}$$

where ζ is the wave impedance, c is the speed of light, $R = |\boldsymbol{r} - \boldsymbol{r}'|$ is the distance between the observation point $\boldsymbol{r}$ and the source point $\boldsymbol{r}'$, and S is the surface of conductors. In addition, the primes denote functions or operators with respect to the observation point $\boldsymbol{r}'$.

Hereinafter we assume that the basis functions are real and independent of the frequency. By expanding the exponential functions in Eq. (2) into the Taylor series, we get the Laurent series expansion of Z_{mn} as follows:

$$Z_{mn} = \sum_{i=-1}^{\infty} s^i Z_{mn}^{(i)}, \tag{3}$$

where the coefficients for the respective powers are expressed as follows:

$$Z_{mn}^{(-1)} = \frac{\zeta c}{4\pi}\int_S \int_S [\nabla \cdot \boldsymbol{F}_m(\boldsymbol{r})] \cdot [\nabla' \cdot \boldsymbol{F}_n(\boldsymbol{r}')]\frac{1}{R}dS'dS,$$

$$Z_{mn}^{(0)} = 0,$$

$$\begin{aligned} Z_{mn}^{(i)} = \frac{(-1)^{i-1}\zeta}{(i-1)!\,4\pi c^i}\int_S \int_S \boldsymbol{F}_m(\boldsymbol{r}) \cdot \boldsymbol{F}_n(\boldsymbol{r}')R^{i-2}dS'dS \\ +\frac{(-1)^{i+1}\zeta}{(i+1)!\,4\pi c^i}\int_S \int_S [\nabla \cdot \boldsymbol{F}_m(\boldsymbol{r})] \cdot [\nabla' \cdot \boldsymbol{F}_n(\boldsymbol{r}')]R^i dS'dS, \end{aligned}$$

where $i \geq 1$. It is notable that several low-order terms in Eq. (3) have explicit physical meanings. The lowest-order term $s^{-1}Z_{mn}^{(-1)}$ is equivalent to the impedance of the capacitance

$$C = \frac{1}{Z_{mn}^{(-1)}} = \left\{\frac{\zeta c}{4\pi}\int_S \int_S [\nabla \cdot \boldsymbol{F}_m(\boldsymbol{r})] \cdot [\nabla' \cdot \boldsymbol{F}_n(\boldsymbol{r}')]\frac{1}{R}dS'dS\right\}^{-1}.$$

Similarly, the term $sZ_{mn}^{(1)}$ is equivalent to the impedance of the inductance

$$\begin{aligned} L = Z_{mn}^{(1)} = \frac{\zeta}{4\pi c}\int_S \int_S \boldsymbol{F}_m(\boldsymbol{r}) \cdot \boldsymbol{F}_n(\boldsymbol{r}')\frac{1}{R}dS'dS \\ +\frac{\zeta}{8\pi c}\int_S \int_S [\nabla \cdot \boldsymbol{F}_m(\boldsymbol{r})] \cdot [\nabla' \cdot \boldsymbol{F}_n(\boldsymbol{r}')]R\,dS'dS. \end{aligned}$$

If the structures of conductors are complicated, a large number of basis functions are required to expand the current distributions on conductors. This results in a large-scale equivalent circuit. To reduce the scale of equivalent circuit, it is useful to expand the current distributions into a small number of modal currents of conductors.

Here, a problem involving a sole conductor is described to show the concept of the eigenmode analysis for the IEM. The self-/mutual impedances, the current coefficients, and the voltage coefficients in Eq. (1) are rewritten in a matrix/vector form as follows:

$$\overline{\boldsymbol{Z}} = \sum_{i=-1}^{\infty} s^i \overline{\boldsymbol{Z}}^{(i)} = \sum_{i=-1}^{\infty} s^i \begin{bmatrix} Z_{11}^{(i)} & \cdots & Z_{1N}^{(i)} \\ \vdots & \ddots & \vdots \\ Z_{N1}^{(i)} & \cdots & Z_{NN}^{(i)} \end{bmatrix},$$

$$\boldsymbol{I} = [I_1 \quad \cdots \quad I_N], \quad \boldsymbol{V} = [V_1 \quad \cdots \quad V_N].$$

If the analysis object is electrically very small, $\overline{\boldsymbol{Z}}$ can reasonably be approximated only by the components that are proportional to s^{-1} and s. Thus, by using $\overline{\boldsymbol{Z}} \simeq s^{-1}\overline{\boldsymbol{Z}}^{(-1)} + s\overline{\boldsymbol{Z}}^{(1)}$, Eq. (1) can be rewritten as follows:

$$s^{-1}\overline{\boldsymbol{Z}}^{(-1)}\boldsymbol{I} + s\overline{\boldsymbol{Z}}^{(1)}\boldsymbol{I} = \boldsymbol{V}, \tag{4}$$

When s and $\boldsymbol{I}$ have specific values, the first and second terms on the left-hand side in Eq. (4) cancel each other and resonate. To obtain such s and $\boldsymbol{I}$, by letting $\boldsymbol{V} = 0$ in Eq. (4), we get the following generalized eigenvalue problem:

$$\overline{\boldsymbol{Z}}^{(-1)}\boldsymbol{I} = \lambda\overline{\boldsymbol{Z}}^{(1)}\boldsymbol{I}, \tag{5}$$

where $\lambda = -s^2 = \omega^2$ is the eigenvalue; i.e., the eigenvalue is the square of the resonant angular frequency. N sets of λ and $\boldsymbol{I}$ satisfy the following orthogonality:

$$\boldsymbol{I}_m^T\overline{\boldsymbol{Z}}^{(-1)}\boldsymbol{I}_n = \delta_{mn}Z_n^{(-1)}, \tag{6}$$

$$\boldsymbol{I}_m^T\overline{\boldsymbol{Z}}^{(1)}\boldsymbol{I}_n = \delta_{mn}Z_n^{(1)}, \tag{7}$$

where $\boldsymbol{I}_m$ and $\boldsymbol{I}_n$ are the mth and nth modal current vectors, respectively, and δ_{mn} is the Kronecker delta. This physically means that there is no capacitive and inductive coupling between modal currents.

By using the elements of the nth modal current vector $\boldsymbol{I}_n = [I_{1n} \cdots I_{Nn}]^T$, the corresponding modal current $\boldsymbol{J}_n(\boldsymbol{r})$ can be expressed as follows:

$$\boldsymbol{J}_n(\boldsymbol{r}) = \sum_{m=1}^{N} I_{mn}\boldsymbol{F}_m(\boldsymbol{r}).$$

By using the modal currents, the current distributions on conductors are expanded as follows:

$$\boldsymbol{J}(\boldsymbol{r}) = \sum_{n=1}^{M} I_n'\boldsymbol{J}_n(\boldsymbol{r}),$$

where I_n' is the nth modal current coefficient, and M is the number of considered modes. It is expected that the current distributions can be approximated only by a small number of modal currents, the resonant frequencies of which are close to the frequency of interest.

In problems involving multiple conductors, such as WPT systems, the eigenmode analysis described above is carried out for each conductor, and the current distributions of each conductor are expanded into several dominant modal currents.

3. Application to a WPT system

Fig. 1 shows the WPT system used in this work. Each Tx and Rx side consists of a feeding loop and a resonance coil, and they are symmetrically placed with respect to the xy-plane of $z = 0$. In addition, $a = 0.8$ mm is the radius of the wires, $b_0 = 93$ mm is the radius of the feeding loops, $b = 118$ mm is the radius of the coils, $c = 8$ mm is the pitch of the coils, $d_0 = 280$ mm is the distance between the Tx and Rx loops, $d = 200$ mm is the distance between the Tx and Rx coils, and $N = 10$ is the number of turns of the coils. In addition, the conductivity of the wires is $\sigma = 58$ MS/m. These parameters were determined such that the transmission coefficient $|S_{21}|$ in the 50-Ω system is maximum at around 13.56 MHz.

Fig. 2 plots the first to the fourth modal currents, the resonant frequencies of which are in ascending order. Here, the modal currents are normalized such that the mean square values are $1/2$. The resonant frequencies of the respective modal currents can be obtained as $f = \sqrt{\lambda}/(2\pi)$, and their values are $f_1 = 13.21$ MHz, $f_2 = 33.58$ MHz, $f_3 = 54.16$ MHz, and $f_4 = 75.46$ MHz, respectively. Among them, the fundamental mode, with a resonant frequency of $f_1 = 13.21$ MHz, is dominant in the frequency range for the WPT system considered in this work.

Fig. 3 plots the frequency dependences of (a) the reflection coefficient $|S_{11}|$, (b) the transmission coefficient $|S_{21}|$, (c) the radiation loss P_r, and (d) the conduction loss P_c. Here, the available power of the Tx port is assumed to be 1 W. In addition, the Laurent series of the self-/mutual impedance is approximated only by the lowest- to fourth-degree terms. In Fig. 3, "Direct" indicates the direct solution obtained without the eigenmode analysis, and its validity has been already confirmed in [7]. On the other hand, "Modal" indicates the modal solution considering only the fundamental mode. According to the results, the behavior of the system can be almost approximated only by the fundamental mode. Incidentally, if the maximum number of modes is $M \geq 2$, both the direct and modal solutions perfectly agree with each other even in the range of $|S_{11}| < -40$ dB. However, in fact, such a slight difference has little impact on the system. Therefore, the required number of modes is concluded to be $M = 1$ in this case.

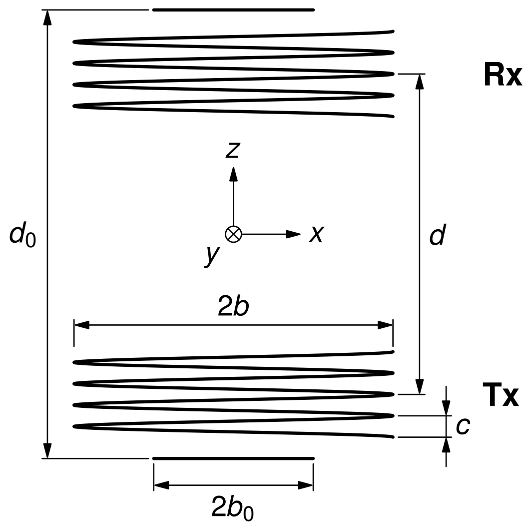

Fig. 1. WPT system.

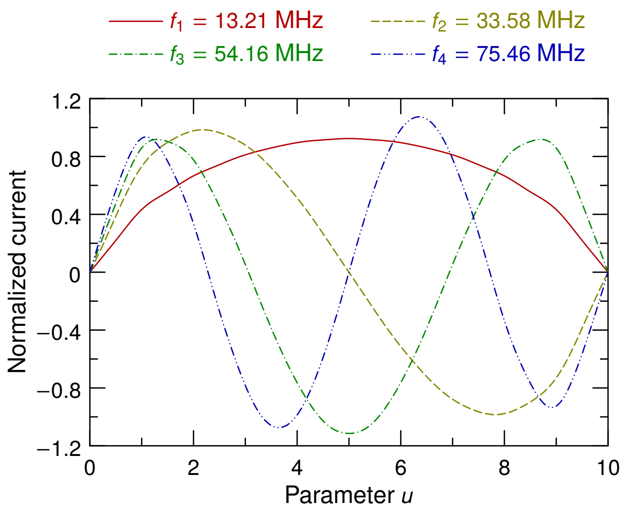

Fig. 2. Modal currents of the coil.

Because the currents of the coils can be approximated only by the fundamental mode, the behavior of the WPT system can be represented by the equivalent circuit model shown in Fig. 4, where V_0 is the electromotive force of the Tx port, and $R_0 = 50\ \Omega$ is the input/output impedance of the ports. Because the equivalent-circuit parameters are expressed by using the self-/mutual impedances between the modal currents, here we describe their notational convention. First, the modal currents of the respective elements are denoted such that: $\boldsymbol{J}_{T0}(\boldsymbol{r})$ is the modal current of the Tx loop; $\boldsymbol{J}_{T1}(\boldsymbol{r})$ is the fundamental modal current of the Tx coil; $\boldsymbol{J}_{R0}(\boldsymbol{r})$ is the modal current of the Rx loop; and $\boldsymbol{J}_{R1}(\boldsymbol{r})$ is the fundamental modal current of the Rx coil. In addition, the self- and mutual impedances between the modal currents are denoted such that: $Z_{T0,T0}$ is the self-impedance of $\boldsymbol{J}_{T0}(\boldsymbol{r})$; $Z_{T1,R1}$ is the mutual impedance between $\boldsymbol{J}_{T1}(\boldsymbol{r})$ and $\boldsymbol{J}_{R1}(\boldsymbol{r})$, and so on. Furthermore, in the Laurent series expansion of the self-/mutual impedance, the coefficient for s^i is denoted by a superscript (i) such as $Z_{T1,R1}^{(i)}$.

The capacitances in Fig. 4 are expressed as follows:

$$C_{T1} = C_{R1} = \frac{Z_{T1,T1}^{(-1)} - Z_{T1,R1}^{(-1)}}{Z_{T1,T1}^{(-1)} Z_{R1,R1}^{(-1)} - Z_{T1,R1}^{(-1)} Z_{R1,T1}^{(-1)}}, \quad C_{T1,R1} = \frac{Z_{T1,R1}^{(-1)}}{Z_{T1,T1}^{(-1)} Z_{R1,R1}^{(-1)} - Z_{T1,R1}^{(-1)} Z_{R1,T1}^{(-1)}}.$$

The inductances in Fig. 4 are expressed as follows:

$$L_{Tm,Tn} = L_{Rm,Rn} = Z_{Tm,Tn}^{(1)}, \quad L_{T1,R1} = Z_{Tm,Rn}^{(1)}, \quad m,n = 0,1.$$

The dependent voltage sources represent the voltage drops due to the modal impedance components that are proportional to s^2, s^3, and s^4; these are expressed as follows:

$$\Delta V_{Tm} = \sum_{n=0}^{1}\sum_{i=2}^{4} s^i Z_{Tm,Tn}^{(i)} I_{Tn} + \sum_{n=0}^{1}\sum_{i=2}^{4} s^i Z_{Tm,Rn}^{(i)} I_{Rn},$$

$$\Delta V_{Rm} = \sum_{n=0}^{1}\sum_{i=2}^{4} s^i Z_{Rm,Tn}^{(i)} I_{Tn} + \sum_{n=0}^{1}\sum_{i=2}^{4} s^i Z_{Rm,Rn}^{(i)} I_{Rn},$$

The impedances $Z_{T0,T0}^{c} = Z_{R0,R0}^{c}$ and $Z_{T1,T1}^{c} = Z_{R1,R1}^{c}$ are ones due to the surface impedance of wires and represent the conduction loss.

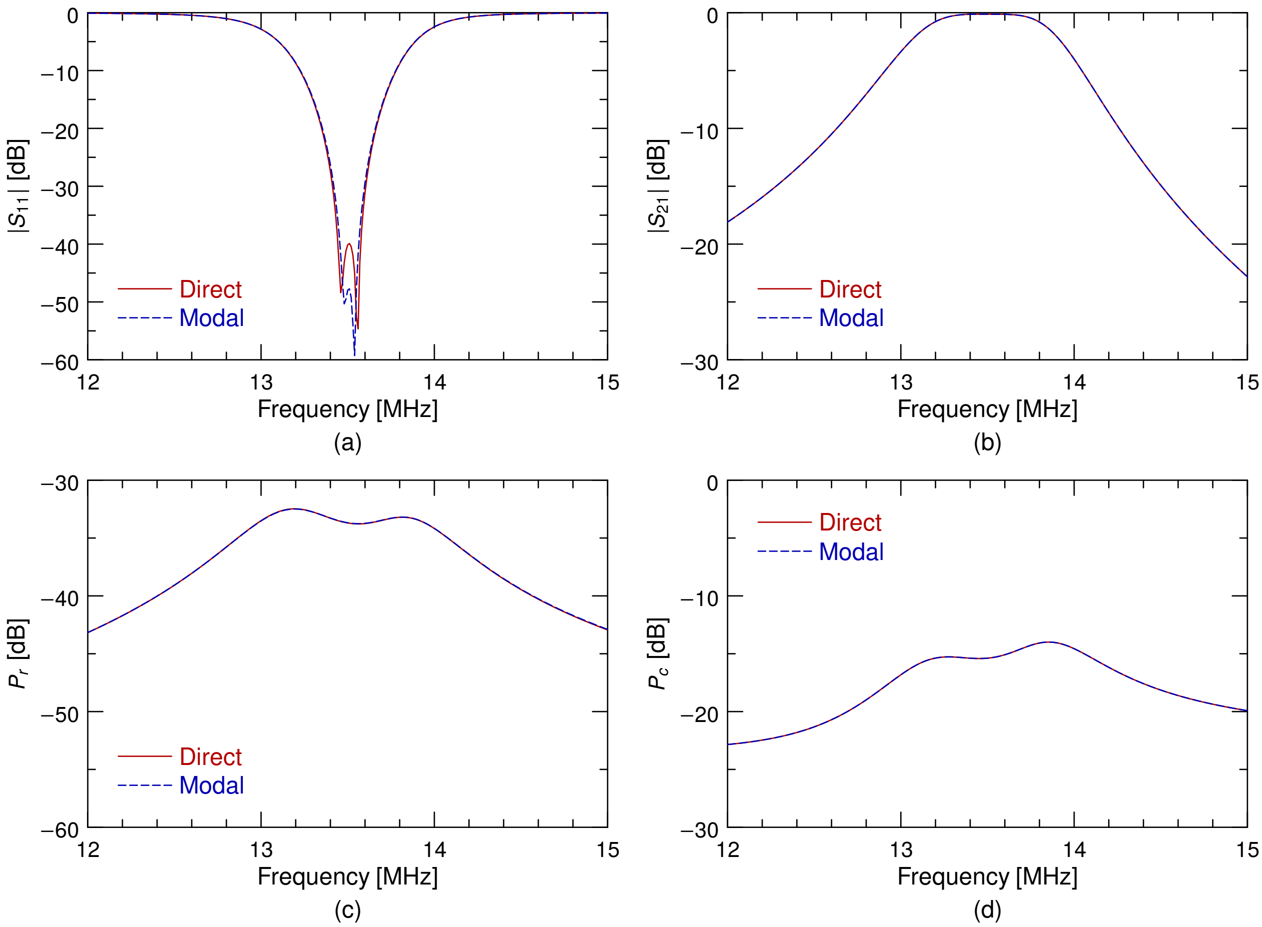

Fig. 3. Frequency dependences of (a) $|S_{11}|$, (b) $|S_{21}|$, (c) P_r, and (d) P_c.

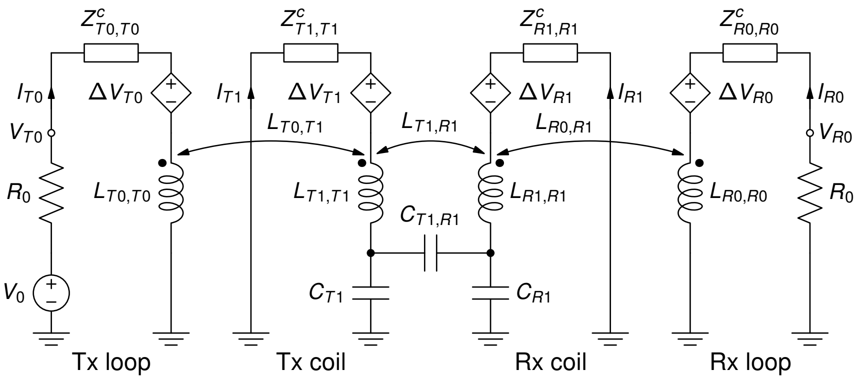

Fig. 4. Equivalent circuit of the WPT system.

4. Conclusion

This paper describes the basic concept of the IEM, and then shows an example application to a wireless power transfer system. In the further studies, the author will apply the IEM to the circuit modeling of intrabody communication channels.

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