

## Waveform Generation with Side-Band Noise by Fourier Transform and Amplitude Modulation

Mei Okajima<sup>1, a</sup>, Hiroshi Sunaga<sup>1</sup>, Takafumi Ogawa<sup>1</sup>, and Mitsuru Shinagawa<sup>1</sup>

<sup>1</sup>Faculty of Science and Engineering, Hosei University,  
3-7-2 Kajino, Koganei, Tokyo 184-8584, Japan

<sup>a</sup><mei.okajima.7p@stu.hosei.ac.jp>

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### Abstract.

This paper describes waveform generation method with a flicker noise on side band spectra by using fast Fourier transform (FFT) and amplitude modulation. The flicker noise is modeled in frequency domain data and is converted into time domain data by using FFT. We try to superimpose the flicker noise on a sinusoidal wave by amplitude modulation. The obtained waveform was examined by spectral analysis by FFT. It was confirmed that the flicker noise exists at side band of the sinusoidal wave spectrum.

### 1. Introduction

We have reported a noise analysis for an optical sensor system with a white noise in previous work [1]-[3]. In general, the sensor system has not only white noise but also other noises which depend on frequency such as a flicker noise [4]. The flicker noise is modeled in frequency domain data and is converted into time domain data by using fast Fourier transform [5] (FFT). We try to superimpose the flicker noise on side band spectra of a sinusoidal wave by amplitude modulation between time domain data of the sinusoidal wave and the flicker noise.

### 2. Waveform Generation with Noise

A schematic of a waveform generation with noise is shown in Fig. 1. The noise power spectra  $P_N(f)$  and phase  $\varphi(f)$  in frequency domain are converted into time domain voltage data  $V_N(t)$  by an inverse FFT ( $\text{FFT}^{-1}$ ). A sinusoidal wave is amplitude modulated by the  $V_N(t)$  with  $V_{DC}$ .

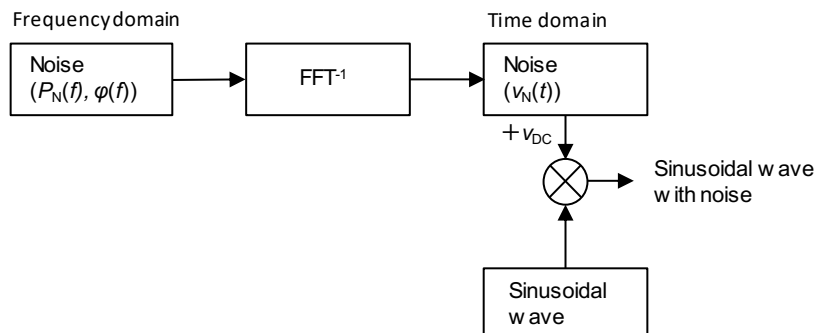


Fig. 1. Schematic of waveform generation with noise.

Waveform  $V_{AM}(t)$  generation with noise is obtained by amplitude modulation as expressed in

$$V_{AM}(t) = (V_{DC} + V_N(t))A \sin(2\pi ft) \quad (1)$$

according to Fig.1. Here,  $f$  is frequency,  $t$  is time, and  $A$  is amplitude. A flicker noise [4] is used as a frequency-dependent noise in this paper. The power of the flicker noise is inversely proportional to frequency.  $P_N(f)$  including the flicker noise  $P_{FN}(f)$  and a white noise  $P_{WN}(f)$  is defined by

$$\begin{aligned} P_N(f) &= P_{FN}(f) + P_{WN}(f) \\ &= (P_{FN0} + Q_1 \sigma_1) / f + P_{WN0} + Q_2 \sigma_2. \end{aligned} \quad (2)$$

Here,  $P_{FN0}$  and  $P_{WN0}$  are average powers of the flicker noise and the white noise.  $Q_1$  and  $Q_2$  are random numbers ( $-1 < Q_1, Q_2 < 1$ ) accordance with Gaussian distribution.  $\sigma_1$  and  $\sigma_2$  are strength standard deviations of the flicker noise and the white noise. A calculated  $P_N(f)$  is shown in Fig. 2 at  $P_{FN0}$  of 9.8  $\mu$ W,  $P_{WN0}$  of 3.3 pW,  $\sigma_1$  of 0.13 mW, and  $\sigma_2$  of 0.42 pW. A number of data is 32768 ( $= 2^{15}$ ), and a frequency step is 500 Hz. The frequency step corresponds to a resolution band width. Characteristics of the flicker noise are found in frequency region of less than 3 MHz, and those of the white noise are found in frequency region of more than 3 MHz.

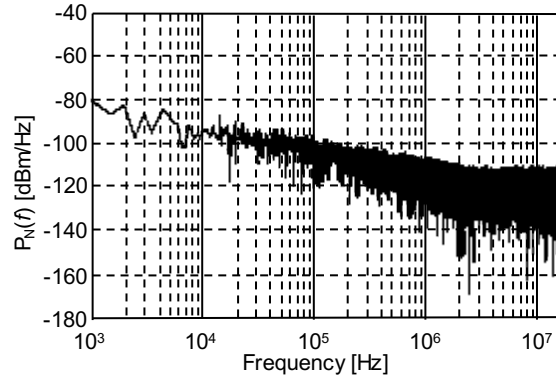


Fig. 2. Modeled power spectra of flicker noise.

$P_N(f)$  based on Eq. (1) is transformed time domain data  $V_N(t)$  by using  $\text{FFT}^{-1}$ . Fig. 3 (a) shows  $V_{N(F+W)}(t)$  with the flicker noise and the white noise, and Fig. 3 (b) shows  $V_{N(W)}(t)$  with the white noise. Slow amplitude fluctuation is found in Fig. 3 (a). It is confirmed that the noise generation method of the flicker noise and the white noise is proper.

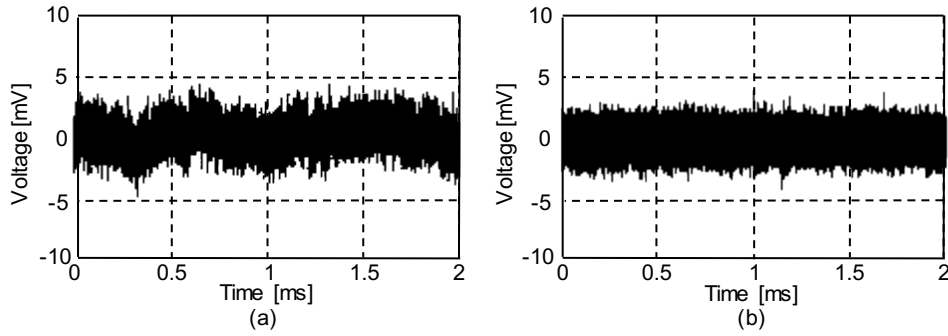


Fig. 3. Waveforms (a) flicker noise and white noise  $V_{N(F+W)}(t)$ , (b) white noise  $V_{N(W)}(t)$ .

### 3. Side Band Noise

The sinusoidal wave with noise based on Eq. (1) and Eq. (2) is calculated. The sinusoidal wave  $V_{AM}(t)$  in time domain is shown in Fig. 4 (a) at  $V_{DC}$  of 0.01 V,  $f$  of 1 MHz, and  $A$  of 1 V. Fig. 4 (b) is zoom-in waveform of Fig. 4 (a) in time range from 0 to 5  $\mu$ s. It is found that there is an influence of the flicker noise and the white noise on the sinusoidal wave.

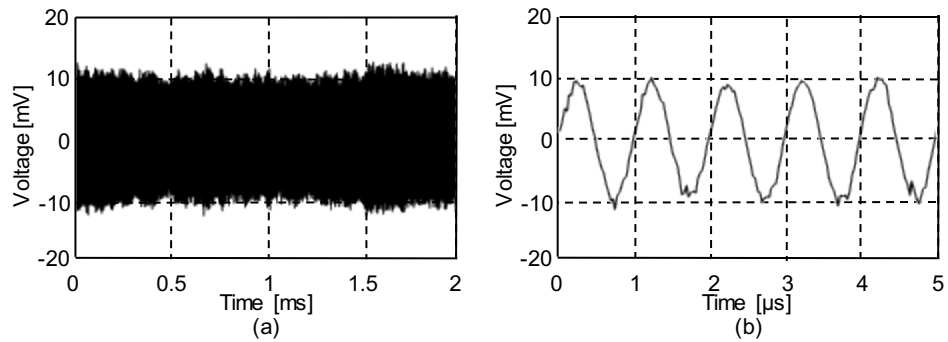


Fig. 4. Sinusoidal wave with noise in (a)  $0 < t < 2$  ms and (b)  $0 < t < 5$   $\mu$ s.

Fig. 5 (a) shows the spectra of the sinusoidal wave with the flicker noise and the white noise, and Fig. 5 (b) shows that with the white noise. The spectra characteristics of Fig.5 (a) are spread by the flicker noise. It is confirmed that the noise can be superimposed into the sinusoidal wave by our waveform generation method with the noise.

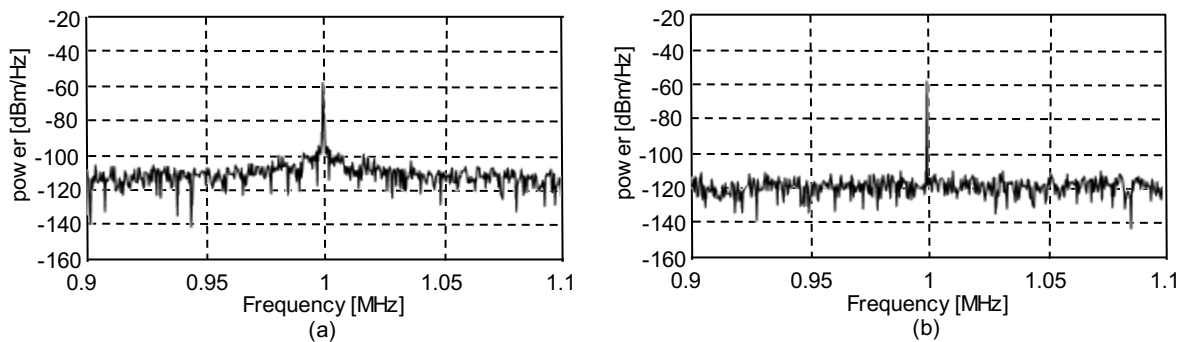


Fig. 5. Spectra of sinusoidal wave (a) with flicker noise and white noise, and (b) with white noise.

#### **4. Conclusion**

We propose a waveform generation method with a noise by using fast Fourier transform (FFT) and amplitude modulation. A flicker noise is modeled in frequency domain data and is converted into time domain data by using FFT. The flicker noise is superimposed on a sinusoidal wave by amplitude modulation. It is confirmed that the noise exists at side band spectra of the sinusoidal wave.

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